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# Sensitivity Analysis of the Texas A & M Wheat (TAMW) Model

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ABSTRACT

Plant simulation models are being investigated for use in forecasting large-area crop yields. The analysis includes a determination of the relative sensitivity of model outputs such as grain yield to changes in input variables and fixed model parameters and functions. Sensitivity information indicates the relative accuracy required for data inputs and aids model refinement.

Response surface methods were used to quantify the relative sensitivity among several factors for the Texas A&M Wheat (TAMW) simulation model. The analysis (for Manhattan, Kansas conditions) shows that among the input variables, daily temperature is the most important determinant of yield when seasonal rainfall is at or above average. The water-related variables become the most critical for drier than normal growing conditions. Some quadratic and interaction effects are found to be statistically significant.

The sensitivity analysis for selected model parameters and functions identifies several influential functions which should be concentrated on for improved model yield prediction accuracy.

Keywords: large-area yield forecasting, central composite design, quadratic response surface, sensitivity measures.

This paper was prepared for limited distribution to the research community outside the U. S. Department of Agriculture. The views expressed herein are not necessarily those of SRS or USDA.

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## INTRODUCTION

The Statistical Reporting Service (SRS), U. S. Department of Agriculture is investigating the use of plant simulation models to forecast large-area crop yields. Present SRS forecasting methods perform very well after the crop has begun producing fruit that can be destructively sampled. However, for earlier plant growth stages, the methodology depends on historic averages and regression relationships that are not very responsive to the current season environmental characteristics which largely determine the final grain yield. Improved early-season yield forecasts are needed, particularly in atypical years, for decision makers in Government, agribusiness, and farm management.

SRS is evaluating several plant models for large-area yield forecasting use. Once initial validation has been conducted on a plot level to demonstrate adequate ability to estimate plant phenology and yield, sensitivity analysis is performed. The sensitivity analysis assesses the relative sensitivity of model response to variations in data inputs and internal model parameters and functions. In this context, sensitivity information serves two main purposes. The first is to help determine the accuracy and resolution required for model inputs in a large-area application. Second, identification of the most influential parameters and functions in the model tells the model developer where to concentrate efforts to improve the accuracy of the simulation.

The sensitivity analysis is done in two steps. The first step is to use a factor screening technique to reduce the number of factors for which sensitivity information will be obtained. Screening is helpful when the number of potentially influential factors is large (say, greater than 50) as it is in a plant model. The second step of the analysis is to fit response surfaces to subsets of the factors selected in the first step. The fitted surfaces approximate the model response within the range of selected factor perturbation. Sensitivity measures are computed from the response surfaces.

The purpose of this report is to document the second step of the sensitivity analysis of the Texas A&M wheat (TAMW) model

(12). The main body of the report is divided into four sections. The first section briefly provides background information on plant simulation models in general and TAMW in particular. The second section indicates some of the factor screening alternatives and references earlier work done in the first step of the analysis. The third section describes the development of the response surface methods used in the sensitivity analysis of TAMW. The intent is to provide some background in the literature, indicate alternatives, and justify the various choices. The fourth section describes the specific application of the response surface methods to TAMW and summarizes the detailed results of the analysis. The main body of the report is followed by a summary (which contains the major conclusions from the analysis), a list of cited references, and ten appendices. The first appendix contains some theory pertaining to a modification in the central composite design. The remaining appendices list the data and analysis of variance tables for the response surface experiments done on TAMW.

## PLANT MODEL BACKGROUND

Plant simulation models attempt to describe the growth and development of a plant mathematically. The level of detail varies among models but can be broadly described as empirical, mechanistic, or some combination of the two. While strictly empirical models are common, so-called mechanistic or process-oriented models all contain some empiricism. Thus, the more complex models describe a number of major plant processes but still rely on experimentally observed relationships where the specific mechanisms are not well understood. Depending on the level of detail, a plant model may contain several thousand lines of computer source code.

The plant models being considered by SRS vary in complexity but can be generally characterized as deterministic rather than stochastic and dynamic as opposed to static. The time increment of the simulation is daily in all cases although some processes within the models may actually operate on shorter or longer increments. A certain amount of initial input data is required at or before the planting date. This typically consists of sowing, soil, location, and soil water information. Daily input requirements include precipitation, maximum and minimum air temperature, and total surface solar radiation. The output generated by the plant simulation varies among models but usually includes soil water information, occurrence dates of various stages of growth, yield components, and final grain yield.

The TAMW model simulates the growth and development of an individual winter wheat plant. Relative to the other models being evaluated by SRS, TAMW is one of the least complex relying much more on empirical relationships than mechanistic. In contrast to the other models, the photosynthesis process is not used to accumulate dry matter. Rather, a competition factor computed from a canopy light interception sub-model limits growth. This approach to some extent replaces the carbohydrate stress factor used in some of the other plant models. TAMW does consider water stress but not nutrient stress. Roots are simulated only within the context of the soil water sub-model to establish the effective depth of the soil water profile. Root location and mass are not considered. Winterkill, a factor which may be very important in some climates, is also not included in the model.

The TAMW program consists of about 950 lines of Fortran source code and averages approximately 4 seconds of cpu time on an IBM 3033 computer. The cpu time can vary widely on different data sets depending on the amount of tillering. In extreme cases, TAMW may take 6 or 7 seconds of cpu time. TAMW has a relatively short main program of 200 lines with 14 subroutines and 3 real functions. There are two main loops in the program. The outside loop makes daily computations and the inside loop keeps track of individual shoots (tillers). Within the shoot loop there is another loop which identifies floret numbers on individual spikelets. The spikelet loop is what makes the execution time of TAMW sensitive to tiller numbers. Additional information can be found in the documentation (12).

## FACTOR SCREENING

A two-step sensitivity analysis has been used because of the large number of potentially critical inputs, parameters, and functions (referred to collectively as "factors") in TAMW. The first step consists of screening to select the factors which have the most influence on the responses of interest. There are a number of factor screening techniques in use. Some use designed experiments such as $2^{k-p}$ designs, random designs, super saturated designs, and group screening designs to estimate factor effects (6, 7, 14, 18). Factors have also been selected with multiple linear regression by using a variable selection technique or by ranking the absolute values of the standardized regression coefficients (4, 5, 19). The choice of technique depends in part on the number of factors and the cost and/or time involved in obtaining the necessary data points. All the above techniques directly use values obtained for at least two different levels of each <u>factor</u>. In contrast, another alternative is path coefficient analysis (20) which was used to identify the most influential model <u>variables</u> and <u>subroutines</u> in TAMW (9). This information aids in the selection of factors since the size of the model is effectively reduced. In addition to automated techniques, the model developer should be consulted so that the factors he feels are important will be included in the analysis.

## RESPONSE SURFACE METHODOLOGY

There are many ways to investigate the sensitivity of model response to changes in the levels of selected factors. The traditional approach is to perturb the factors individually and observe the model response. The sensitivity can be quantified by finding the value when relative change in response divided by relative change in the factor is a maximum. (See (8) for an example of this procedure.) There are two drawbacks of perturbing factors individually. Many of the factors are interrelated and there is no way to estimate possible interaction. Secondly, a large number of model runs are needed to adequately estimate the individual curves when there are many potentially influential factors. Baker and Bargmann (1) suggested that response surface techniques be used to alleviate these problems.

The usual application of response surface methodology is to determine the particular combination of factor levels that optimizes a certain response (see (16) for general reference). However, optimization is not of concern in a sensitivity analysis. Rather, the fitted response surface is used to approximate model behavior within a selected range of factor levels. Several measures can be computed from the fitted surface to quantify sensitivity. These will be discussed later.

### Functional Choices

The choice of functions for fitting a response surface is large. Within a biological context, Mead and Pike (13) delineate four main groups: polynomial, rectangular hyperbolae, inverse polynomial, and exponential. Of these, polynomials of degree p are the most frequently used because of their relative simplicity and the fact that parameter estimates using ordinary least squares (OLS) are straightforward (13). Disadvantages of using the polynomial as a response function include questionable biological justification and unreliable extrapolation outside of the factor space (13). A specific disadvantage of the quadratic polynomial is that it is symmetric about the optimum (13). For our purposes, biological justification is unimportant and the factor space is carefully selected so that extrapolation is unnecessary. The rather unrealistic constraint of symmetry in the quadratic polynomial is not a problem because typically the global optimum lies outside the factor space so that assymmetry about the center point can be accommodated. Usually the response within the factor space forms a ridge in which the local optimum is somewhere on the boundary of the factor levels. Polynomial functions were chosen for this study because of ease in application and interpretation.

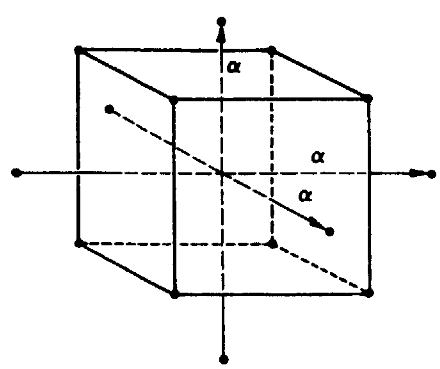
### Selecting the Order of Polynomial

The simplest polynomial is, of course, the linear model ($p = 1$). While first-order interactions can be estimated, there is risk of bias in the intercept and linear terms. For example, if the true underlying relationship is in fact quadratic, the intercept is biased by the quadratic terms while the linear coefficients remain unbiased (16). If the true relationship is cubic, the linear coefficients are biased by certain cubic terms as well (16). Because of the use of numerous non-linear functions in plant simulation models, there is good reason to expect some of the factor-response relationships to be curvilinear. How high a degree of polynomial to use becomes a question of cost. Practically speaking, the only realistic choices are the quadratic and cubic. Depending on the design, if there are three factors in the experiment, a cubic response surface requires about twice as many points (model runs) as the quadratic. For 6 factors, the cubic would require more than 10 times as many points. In the absence of compelling evidence to do otherwise, a quadratic response is assumed for this study. Unfortunately, it is not possible to test for cubic lack of fit in a second-order model when the response data are generated by a deterministic plant model.

## Experimental Design

Experimental designs for fitting a second-order response surface must involve at least three levels of each factor to permit estimation of all the effects. The design which is immediately suggested is the $3^k$ factorial. However, as the number of factors (k) increases, the number of design points gets very large. As an alternative to the $3^k$ factorial, Box and Wilson (3) proposed an extremely useful class of designs called composite designs of which the central composite design (CCD) is a special case. Though many other designs have been proposed since 1951, the CCD continues to perform the best for quadratic surfaces at many values of k and it was selected for this analysis. See (11) and (17) for a comparison of the CCD to several different design alternatives.

When k=2, the CCD is a $3^2$ factorial and for increasing k, the number of design points becomes substantially less than the full $3^k$ factorial. The basic CCD consists of a full $2^k$ factorial (called "lattice" points) plus 2k "axial" points and n "center" points. For the k=3 case, the CCD can be visualized as in the following figure.



The vertices of the cube are the lattice points of the $2^3$ factorial. The axial points lie at a distance $\alpha$ from the center point along each of the three axes.

**Characteristics of the CCD**

The values of $\alpha$ and n determine various characteristics of the CCD. Four characteristics will be discussed briefly: rotatability, orthogonality, uniform-precision, and minimum-bias. A design is said to be rotatable if the variance of the estimated response y at any point on the fitted surface is a function of the distance from the point to the design center and not the direction. Orthogonality provides uncorrelated parameter estimates with minimum variance. Uniform-precision requires that the variance of y at the design center be the same at some arbitrary distance from the center, usually the lattice points. Minimum-bias designs have minimum bias of y and provide some protection from the presence of higher order terms.

In most applications of the CCD, a distance $\alpha$ for the axial points is determined (based on k) that provides a rotatable design and each of the other characteristics may be obtained by adding an appropriate number of center points. However, when experiments are conducted on a deterministic plant model, there is no "pure error" because repeated points provide identical responses. Multiple center points can still be used to obtain desired design characteristics, but then the "lack-of-fit" mean square should be used for hypothesis testing rather than the total mean square error. Otherwise, the fitted surface will appear to be estimated with greater precision than possible.

A simulation study by Montgomery and Evans (15) and some additional testing of our own suggests that an orthogonal (nonrotatable) CCD with a single center point may be preferable in the present situation. Montgomery and Evans evaluated several designs over a range of known surfaces in an optimization situation to see which performed the best according to two of their criteria. They state that in a situation with no random error, the orthogonal rotatable CCD was preferred over the uniform-precision or the minimum-bias rotatable CCD. They also state that in an optimization problem, rotatability is always worthwhile. However, whether the variance of y is independent of direction is not of primary importance when optimization is not being sought. In a sensitivity analysis, the bias, precision, and independence of the parameter estimates is of main concern. We did a limited amount of testing to determine whether an orthogonal or rotatable CCD with a single center point would be preferred when a quadratic is used in the presence of a known cubic function. In general, the orthogonal CCD had slightly less bias and better precision than the rotatable CCD. Also, independence of parameter estimates provided by orthogonality is desirable because otherwise, some of the sensitivity measures might be influenced by interrelationships among the variables. For these reasons, an orthogonal design was selected for this study. Note that even with appropriate selection of $\alpha$, the quadratic parameter estimates are not, in general, orthogonal to the intercept unless they are corrected for their means. When the pure quadratic terms are corrected, the intercept is equal to the mean response over all design points. We did not correct the quadratic terms for their means so, in general, the intercept differs from the response mean.

Two modifications of the basic orthogonal CCD were used to reduce the number of required data points -- fractional replication and blocking. When the number of factors reaches six, for example, a one-half replication of the $2^6$ factorial portion of the CCD was used rather than the full factorial design with a saving of 32 model runs. Using the six-factor interaction as a defining contrast, independent estimates were still made for all main effects and first-order interactions. The second modification was to divide the points in the CCD into orthogonal blocks. This is useful because to actually obtain response points from a plant model, a certain set of daily climatic input values must be specified. Depending on the factors that are selected, the sensitivity measures from the fitted surface may not be independent of the set of daily climatic input. This suggests that experiments be repeated on several different sets of input data. While these experiments can be done separately, the total number of required response points can be greatly reduced by "blocking" the climatic input and running a single experiment. The underlying assumption is that the different climatic scenarios have an additive effect on the response. That is, estimated response surfaces obtained from separate experiments using the different climatic scenarios have similar parameter estimates except for possible differences in the intercept terms. This assumption appears to be realistic with factors that are either fixed parameters or functions in the plant model. Orthogonality of the blocks is desired so that the block effects can be removed independently of the estimated coefficients.

A CCD can be divided into blocks differently depending on the number of factors. For this study the factorial portion of the design was split into two blocks and the axial portion became a third block. Orthogonality of the blocks is obtained by solving a relationship for $\alpha$ and the number of center points in each block ($n_i$) (see Appendix A for details). The solution for $\alpha$ and $n_i$ is not unique so one can choose the $\alpha$ corresponding to the fewest center points. For example, a four-factor orthogonal CCD which blocks orthogonally would have two factorial blocks each consisting of a one-half fraction (i.e. $1/2 \cdot 2^4$ = 8 points) and three center points, and an axial block ($2 \cdot 4$ = 8 points) with no center points and a value of $\alpha = 1.712$. For a deterministic plant model, the repeated center points are identical within a block but different between blocks because of different climatic scenarios. As discussed earlier, significance tests on the parameter estimates and factor effects should be done using the lack-of-fit mean square rather than total error.

The block effects were removed using two uncorrelated covariates which were not crossed with the factor variables in the response surface. The first covariate, F, identifies the three blocks with an arbitrary numbering scheme (e.g. block 1, block 2, and block 3). If F is used as a single covariate, the choice of block numbers has no effect on the parameter estimates (except for very small changes in the quadratic terms) but does influence the intercept and the mean square error. Hence, the arbitrary values of F affect the significance tests of the parameter estimates. To avoid this, a second covariate, FSQ, was included where $FSQ = (F-\bar{F})^2$. With three blocks, different numbering schemes provide identical results except for the intercept term and the loading on F and FSQ.

**Method of Factor Perturbation**

To apply response surface techniques to a plant model, settings for the factor levels must be selected. Sometimes the model developer has information on the reliability of various parameter estimates or functional fits. If so, it should be used to determine the range of perturbation. This information was not available for TAMW so a consistent 20% coefficient of variation (CV) was assumed throughout the analysis. The choice of 20% was somewhat arbitrary but the important thing is to be consistent across all factors unless there is strong evidence to do otherwise. Consistency assures that one factor will not appear more (less) influential than another solely due to arbitrarily varying amounts of perturbation. Another important consideration in selecting settings for the factor levels is to keep the range of perturbation reasonable. In an effort to accomplish this, the axial points were assumed to form a 90% confidence interval (normality also assumed). For example if a fixed model parameter of interest is equal to 2, a 20% CV would imply that the 90% confidence interval (CI) on the parameter would be (1.342, 2.658). Thus, (1.342, 2, 2.658) would correspond to the factor levels ($-\alpha$, 0, $+\alpha$). Depending then on the value of $\alpha$, the lattice points would be scaled accordingly. For example, if $\alpha = 1.5$, the lattice points (-1, +1) would correspond to the values (1.561, 2.439). Two approaches of factor perturbation were taken on functions: one perturbation was a vertical percentage shift in the function (again assuming a 20% CV) and the other was a fixed horizontal shift.

When daily climatic input variables are the factors, historic weather records at a particular location were used to estimate the standard error of seasonal means rather than using an arbitrary 20% CV. The same assumption of a 90% CI at the axial points was used. The daily input values were perturbed by adding certain values over the entire season so as to give the desired seasonal average. Alternatively, perturbation could be done during certain periods of the growing season since the relative effect of climatic variables is known to depend on growth stage. This type of analysis has been done by perturbing factors individually but not by response surface methods (8).

### Sensitivity Measures

Once a surface has been fit for a particular set of factors, various sensitivity measures can be computed. Two measures used in this study are described here. The first is the "relative" sensitivity measure proposed by Baker and Bargmann (1). They suggested that this measure be computed by dividing each estimated coefficient by the intercept. Since in this study the intercept differs from the response mean depending on the quadratic parameter estimates and the blocking covariates (if any), the response mean was used as the divisor rather than the intercept. The larger the value of the sensitivity measure, the greater the effect of the factor on the response. Division by the response mean provides relative values that can be compared among different sets of factors and responses. The relative sensitivity measure was computed only for coefficients determined to be significantly different from zero.

The relative sensitivity measure does not reflect differences in precision among linear, quadratic, and interaction effects. (Lower precision on the quadratic and interaction effects is the "penalty" for using a CCD rather than the $3^k$ factorial design.) A simple alternative is to take the partial sum of squares associated with each factor (linear, quadratic, and interaction) and compute proportions of the sum of all the partial sums of squares. The resulting partial SS sensitivity measure was also used in this study.

## RESPONSE SURFACE EXPERIMENTS ON TAMW

Several response surface experiments were recommended after factor screening with path coefficient analysis (9). All these experiments have factors pertaining to the parameters and functions in TAMW. The first four experiments deal with phenology responses while the last four explore yield responses. Only the latter recommended experiments are presented in this report. In addition, several other experiments have been conducted using the climatic inputs as factors. Again, only yield responses are presented.

The input data for all the experiments are based on conditions at Manhattan, Kansas which is on the eastern side of the major U. S. winter wheat region. The initial inputs used are typical for the Manhattan area and include an October 1 planting date, 8-inch row spacing, density of 189 plants per square meter, and 60% initial plant available soil water. Several parameters were also specified for soils common to the Manhattan area. Daily climatic inputs for TAMW were simulated using a stochastic weather simulation model (10). Three random sequences were generated for the blocks discussed earlier. In addition, a fourth sequence was selected from several other random sequences to provide a more "normal" season. Seasonal statistics for these four climatic scenarios are summarized in Table 1. The length of the growing season varies for the simulated data because the climatic input variables influence when TAMW simulates physiological maturity. (Sensitivity of simulated phenology to changes in climatic inputs is presented in an earlier report (8).) As a basis of comparison, Table 1 also gives summary statistics for 22 years of actual data recorded at Manhattan, Kansas. Statistics for the actual data were initially computed for an assumed October through June growing season which covered 273 days. Since June is typically warmer and wetter than the other months in the growing season, there was a need for summarization of actual data on the same varying lengths of season as in the simulated data. Unfortunately, a "dirty" input tape allowed only one pass through the 22 years of Manhattan data so the statistics for the actual data with varying lengths of growing season are all estimated from the 273-day season. From Table 1, it can be seen that block 1 is drier than average and blocks 2 and 3 are wetter than average. As desired, the "normal" season is very close to the 22-year average.

Factor levels were determined as discussed earlier. In general, axial points for parameter and function factors were chosen as endpoints of 90% confidence intervals (assuming a 20% CV and normality) and lattice points were located at a proportionate distance from the center depending on the value of $\alpha$. For the input factors, the standard errors for the 260-day actual season in Table 1 were used to establish the factor variation instead of a 20% CV. The 260-day season was selected because it is intermediate in length and the same factor levels were desired for all blocks.

TAMW was run once for each factor combination to produce the necessary response data. Quadratic response surfaces were fit using the RSREG procedure in SAS (2). The factor and response data and detailed RSREG output are contained in Appendices B - J. Sensitivity measures computed from the fitted response surfaces are presented in the next section.

Table 1 - Summary data for four simulated seasons of climatic data at Manhattan, Kansas.

| Block | Days per Season | Total Rainfall (inches) | Avg. Max. Temperature (°F) | Avg. Min. Temperature (°F) | Avg. Daily Solar Radiation (L) |
|---|---|---|---|---|---|
| 1 | 260 | 11.67 | 58.6 | 36.4 | 316.3 |
| 2 | 263 | 40.07 1/ | 58.8 | 36.3 | 321.7 |
| 3 | 255 | 23.44 | 58.6 | 36.3 | 321.0 |
| Normal | 262 | 18.86 | 57.3 | 34.6 | 316.4 |
| Actual 2/ | 273 | 21.6(4.7) | 59.4(1.2) | 36.9(1.1) | 327.1(11.0) |
| Estimated Actual 3/ | 255 | 18.2(4.0) | 57.4(1.15) | 34.9(1.03) | 310.3(10.8) |
| | 260 | 19.1(4.2) | 57.9(1.15) | 35.4(1.04) | 314.9(10.8) |
| | 262 | 19.5(4.2) | 58.1(1.15) | 35.6(1.05) | 316.9(10.9) |
| | 263 | 19.7(4.3) | 58.2(1.15) | 35.7(1.05) | 317.9(10.9) |

1/ 7.4 inches of the 40.07 total occurred just before physiological maturity.

2/ Based on 22 years of observed data where the growing season was assumed to cover the months October through June (273 days). The standard errors are in parentheses.

3/ Summary statistics for the actual data were adjusted to agree with the varying lengths of growing seasons in the simulated scenarios.

## Results - Input Variables

A four-factor orthogonal central composite design with three orthogonal blocks was used to estimate the sensitivity of yield response to changes in initial soil water, and daily precipitation, temperature (maximum and minimum), and solar radiation. Maximum and minimum temperature were perturbed jointly with a single factor because they are highly correlated with one another. The blocks constitute the three different climatic scenarios with the implicit assumption that the effect of different seasons is additive. The RSREG results for this experiment (number 0) are contained in Appendix B. Only the regression for the number of heads per plant (Table B4) is significant at the 10% level and generally the precision of the parameter estimates for all yield response variables is low. This is an indication that the assumption of additivity is violated. That is, very different response surfaces exist for the three sets of climatic data and the adjustment for different intercepts using covariates is not adequate. Sensitivity measures are not given for this experiment.

Since it was not acceptable to run a single experiment on input variables with three blocks, separate four-factor experiments were run for each climatic scenario including the "normal" one. These experiments are numbered 1-4 and detailed results are in Appendices C-F. Table 2a gives variable definitions and factor level settings. Table 2b summarizes the sensitivity results for experiments 1-4. As described earlier, the relative and partial sums of squares sensitivity measures are given. Both measures are given in terms of a percentage. The climatic scenarios (experiments) are in the order 1, normal, 3, and 2 which corresponds to driest to wettest. For the relative sensitivity measure, only those based on coefficients which are significantly different from zero at the 5% level are given. For the partial SS measure, significance of the partial SS from zero ($\alpha = .05$) is indicated with an asterisk (*). Experiment 1 was not run for response variable Y3 because, as seen in Appendix C, Table C1, Y3 is essentially the same for all factor combinations.

The results in Table 2b indicate that during the driest season (experiment 1), the yield responses are most sensitive to changes in the initial soil water and daily precipitation. The initial plant available soil water (X1) at the center point (i.e. X1 = 0) is 60%. From Table 2a, when X1 = -1 the corresponding soil water content is 36.7% (60% minus 23.3%) and (from Table 2b) the relative change in the mean yield response (Y4) is -29.1% (-18.0% plus -11.1%). In absolute terms, a decrease of 23.3% in the initial soil water results in a 847 kg/hec decrease in the yield. In terms of the model estimated yield at the center point (i.e. when none of the factors are perturbed), the yield decrease is 24%. When X1 = +1, the resultant change in the center point yield or, more succinctly, the "control" yield is plus 6%. Considering the effect of changing the daily rainfall (X2) in experiment 1, it can be determined from Tables 2a and 2b that when X2 = -1, the control yield decreases 27%. When X2 = +1, the control yield increases 5%. It can also be seen in Table 2b that the initial soil water by daily rainfall interaction is highly significant in experiment 1. When both X1 = -1 and X2 = -1, the control yield decreases 66%. However, when at least one of these two factors is at the high level (i.e. (-1, +1), (+1, -1), or (+1, +1)), the control yield only decreases between 3 and 7%.

For large-area application, these results suggest that unless the seasonal rainfall is less than 11.67 inches and the initial soil water is below 60%, these water-related inputs have a minor effect on the simulated yield. This is so because negative perturbations caused much larger yield changes than positive perturbations. Also, the relationship between initial soil water and rainfall is such that if the initial soil water is at least 83% then the seasonal rainfall can be as low as 8.7 inches without much effect on the model yield. Conversely, if the seasonal rainfall is at least 14.7 inches then initial soil water values as low as 36.7% have little effect on yield.

From an application standpoint, these results are helpful because they give guidelines on when it is necessary to obtain accurate inputs for initial soil water and daily rainfall. For example, if it can be easily determined that a selected field has a high amount of stored soil water at planting then actual soil water does not need to be measured with soil samples and estimates of daily rainfall at the field can be fairly rough if the probability of less than, say, 10 inches of seasonal rainfall is very small. On the other hand, if it is apparent that initial soil water amounts are quite low and the climate is typically dry then it is necessary to estimate the soil water and daily rainfall accurately. The sensitivity analysis shows that for dry conditions a 1% error in either the initial soil water or the seasonal rainfall total results in about a 1% change in the model yield. If both the water-related inputs are in error by 1% on the low side then the model yield would decrease about 2.6%. Note that the estimation error of primary concern on daily rainfall is "systematic" and not "random". In other words, individual daily amounts can be in error by more than 1% without a similar change in yield so long as the errors are not mostly in the same direction.

When seasonal rainfall amounts are average and above for Manhattan, the results in Table 2b show that daily temperature is by far the most critical factor in determining the yield responses in TAMW. Increasing temperature has a positive

Table 2a - Variable definitions and factor level settings for input response surface experiments 1-4.

Response
Variables

Y1 - Average number of grains per head.

Y2 - Average weight per grain (mg).

Y3 - Average number of heads per plant.

Y4 - Average grain yield per hectare (kg).

Factor
Variables

X1 - Initial soil water available to the plant (percentage shift).

X2 - Daily rainfall (percentage shift for entire growing season).

X3 - Daily maximum and minimum temperature (constant shift for entire growing season).

X4 - Daily solar radiation (constant shift for entire growing season).

| | Settings (±) | |
|---|---|---|
| Factor | Lattice | Axial ($\alpha$ = 1.414) |
| X1 | 23.3% | 32.9% |
| X2 | 25.6% | 36.2% |
| X3 | 1.16°F | 1.65°F |
| X4 | 12.6 L | 17.8 L |

Table 2b - Sensitivity measures for input response surface experiments 1-4.

## Relative Sensitivity Measures (%)

| Response | Factor | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Experiment | X1 | X2 | X3 | X4 | $X1^2$ | $X2^2$ | $X3^2$ | $X4^2$ | X1X2 | X2X3 |
| Y1 | | | | | | | | | | |
| 1 | 5.9 | 5.7 | | | -3.6 | | | | -6.6 | |
| Normal | | | -2.2 | | | | 4.4 | | | |
| 3 | | | 12.8 | | | | -2.7 | | | |
| 2 | | | 7.9 | | | | 2.1 | | | |
| Y2 | | | | | | | | | | |
| 1 | 14.7 | 16.5 | | | -7.7 | -10.4 | -2.6 | 2.4 | -13.8 | |
| Normal | | | -0.6 | | | | | | | |
| 3 | 0.5 | 0.5 | -2.2 | | | | 2.1 | | -0.6 | |
| 2 | | | -0.9 | | | | 1.9 | | | |
| Y3 | | | | | | | | | | |
| 1 | | | | | | | | | | |
| Normal | | -1.1 | 9.1 | | | | | | | -1.4 |
| 3 | | | 2.3 | | 0.9 | 0.9 | | 0.9 | | |
| 2 | | -1.7 | 8.0 | | -1.2 | -1.2 | -6.3 | -1.2 | | 2.1 |
| Y4 | | | | | | | | | | |
| 1 | 18.0 | 19.7 | | -1.6 | -11.1 | -13.1 | | | -17.4 | 1.6 |
| Normal | | | 6.5 | | | | 4.9 | | | |
| 3 | | | 13.1 | | 1.4 | 1.4 | | 1.4 | | |
| 2 | | | 15.1 | | | | | | | 1.8 |

## Partial SS Percentage of Total SS

| Response | Factor | | | |
|---|---|---|---|---|
| Experiment | X1 | X2 | X3 | X4 |
| Y1 | | | | |
| 1 | 50.7* | 47.3* | 0.2 | 1.8 |
| Normal | 7.3 | 7.4 | 77.9* | 7.3 |
| 3 | 0.0 | 0.0 | 99.9* | 0.0 |
| 2 | 0.1 | 0.2 | 99.5* | 0.1 |
| Y2 | | | | |
| 1 | 44.9* | 53.8* | 0.6 | 0.7 |
| Normal | 0.2 | 3.3 | 96.3* | 0.2 |
| 3 | 8.8* | 8.8* | 77.5* | 4.9 |
| 2 | 0.3 | 0.4 | 99.1* | 0.3 |

| Response | Factor | | | |
|---|---|---|---|---|
| Experiment | X1 | X2 | X3 | X4 |
| Y3 | | | | |
| 1 | - | - | - | - |
| Normal | 0.0 | 3.4* | 96.6* | 0.0 |
| 3 | 5.2 | 5.2 | 84.4* | 5.2 |
| 2 | 0.6 | 7.6* | 91.1* | 0.6 |
| Y4 | | | | |
| 1 | 46.6* | 52.8* | 0.3 | 0.4* |
| Normal | 2.0 | 6.8 | 89.2* | 2.0 |
| 3 | 0.8 | 0.8 | 97.6* | 0.7 |
| 2 | 0.1 | 2.1 | 97.8* | 0.1 |

*Significant at 5% level.

effect on grain number per head (Y1), head number per plant (Y3), and the grain yield (Y4). Increasing temperature has a negative effect on weight per grain (Y2). For the grain yield response, the effects are linear in experiments 2 and 3. The quadratic response in the "normal" season is due to below average temperature prior to grain filling and above average temperature during grain filling. The temperature effect on grain yield over all four experiments shows a tendency to increase as seasonal rainfall increases. Referring also to Table 2a, it can be seen that a 1.16°F increase in the daily temperature results in a 12% increase in grain yield for the "normal" experiment and 13% and 15% increases in grain yield for experiments 3 and 2, respectively. Experiments 3 and 2 have similar decreases in grain yield when the temperature is decreased by 1.16°F but the "normal" experiment has only a 2% decrease because of the quadratic effect.

For large-area application, these results suggest that the daily maximum and minimum temperature inputs must be estimated with very little systematic error (bias) because if the daily average temperature is biased by just 1°F, the model yield will change roughly 10% depending on seasonal rainfall. However, much larger random errors in the daily temperature estimates can be tolerated. The fact that TAMW uses the daily temperature average to simulate the various plant processes implies that even some bias in estimating the maximum and minimum would not matter if one were fortunate enough to have offsetting biases.

The results in Table 2b show that, as expected, reasonable solar radiation perturbations have very little influence on model yield. This is very helpful from an application standpoint because solar radiation is difficult to measure at the field level on a large scale and a relatively small number of weather stations report daily solar radiation. Thus, fairly large errors in the daily estimates can be tolerated but, again, large biases should be avoided.

## Results - Parameter and Function Variables

Factor variable definitions for experiments 5-8 are in Table 3a. Response variables are the same as for experiments 1-4. Factor level settings for experiments 5-7 are in Table 3b. Experiment 8 is a combination of the most important factors in experiments 5-7 and uses the same factor level settings. The functions that are perturbed in Experiment 8 are shown graphically in Figures 1-5.

In experiments 5 and 7 a four-factor orthogonal CCD with three orthogonal blocks was used. A six-factor orthogonal CCD with a one-half replication of the factorial portion and three orthogonal blocks was used for experiments 6 and 8. The factor combinations, yield responses, and detailed RSREG results are in Appendices G-J. There is no evidence in these results that the additivity assumption used in blocking the three different climatic scenarios into single experiments is violated in this case.

Table 3c summarizes the sensitivity results for experiments 5-8. Experiment 5 examined the influence of the leaf appearance function (X1 and X2), a parameter affecting the rapidity with which leaves appear on new tillers (X3), and a function that determines the number of leaves which appear between the emergence of successive tillers (X4) on the simulated yield. The sensitivity measures clearly show that shifting the horizontal temperature axis of the leaf appearance function (X2) (see Figure 1) has the greatest influence on the yield responses. Most of the change in the grain yield per hectare (Y4) was due to a change in the number of heads per plant (Y3). These four factor variables have no significant influence on the weight per grain (Y2).

Experiment 6 perturbs the duration and rate of spikelet initiation functions (X1, X2, and X3) and the duration and rate of floret initiation functions (X4, X5, and X6). All six factors affect only the grain number per head (Y1) and the grain yield per hectare (Y4) so these two estimated response surfaces are identical. A vertical percentage shift in the rate of floret initiation function (X6) (see Figure 3) has the greatest influence on yield but is followed closely by vertical percentage shifts in the spikelet initiation duration function (X2) (see Figure 2) and the floret initiation duration function (X5).

Experiment 7 perturbs the duration (X1 and X2) and rate (X3 and X4) functions for grain filling. The factors influence only the weight per grain (Y2) and the yield per hectare (Y4) and, hence, the results are identical for these two responses. In this experiment, the yield response is most sensitive to a vertical percentage shift in the grain filling rate function (X4) (see Figure 5). However, shifting the grain filling duration function (X1 and X2) also has a strong influence on yield (see Figure 4).

Experiment 8 combines the most influential factors from experiments 5-7. While this procedure (as opposed to a single 14-factor experiment) fails to estimate all possible interactions among the factors suggested by the path analysis, the grouping of similar factors in experiments 5-7 tends to reduce the possibility of missing highly significant interactions. The magnitude of the linear relative sensitivity measures for the yield response (Y4) was used as a basis for selecting

Table 3a - Factor variable definitions for parameter and function response surface experiments 5-8.

Factor
Variables

F - Block indicator for three different random sequences of climatic data.

FSQ - $(F-\bar{F})^2$

Experiment 5 - WLEAF and TEMERG Subroutines

X1 - Rate of leaf appearance function (vertical percentage shift).

X2 - Rate of leaf appearance function (horizontal constant shift).

X3 - Parameter affecting leaf appearance rate on a new tiller (percentage change).

X4 - Leaf interval between tiller function (vertical percentage shift).

Experiment 6 - FLORET Subroutine

X1 - Duration and rate of spikelet initiation functions (horizontal constant shifts).

X2 - Duration of spikelet initiation function (vertical percentage shift).

X3 - Rate of spikelet initiation function (vertical percentage shift).

X4 - Duration and rate of floret initiation functions (horizontal constant shifts).

X5 - Duration of floret initiation function (vertical percentage shift).

X6 - Rate of floret initiation function (vertical percentage shift).

Experiment 7 - GRFILL Subroutine

X1 - Duration of grain filling function (horizontal constant shift).

X2 - Duration of grain filling function (vertical percentage shift).

X3 - Rate of grain filling function (horizontal constant shift).

X4 - Rate of grain filling function (vertical percentage shift).

Experiment 8 - Combined from 5, 6, and 7

X1 - Factor variable X2 from experiment 5.

X2, X3 - Factor variables X2 and X6 from experiment 6.

X4, X5, X6 - Factor variables X1, X2, and X4 from experiment 7.

**Table 3b - Factor level settings for parameter and function response surface experiments 5-7.**

| | Factor | Settings (±) Lattice | Axial |
|---|---|---|---|
| Experiment 5 ($\alpha$ = 1.712) | X1 | 19.2% | 32.9% |
| | X2 | 2.92°C | 5.00°C |
| | X3 | 19.2% | 32.9% |
| | X4 | 19.2% | 32.9% |
| Experiment 6 ($\alpha$ = 2.228) | X1 | 2.24°C | 5.00°C |
| | X2 | 14.8% | 32.9% |
| | X3 | 14.8% | 32.9% |
| | X4 | 2.24°C | 5.00°C |
| | X5 | 14.8% | 32.9% |
| | X6 | 14.8% | 32.9% |
| Experiment 7 ($\alpha$ = 1.712) | X1 | 2.92°C | 5.00°C |
| | X2 | 19.2% | 32.9% |
| | X3 | 2.92°C | 5.00°C |
| | X4 | 19.2% | 32.9% |

Table 3c - Sensitivity measures for parameter and function response surface experiments 5-8.

## Relative Sensitivity Measures (%)

-- Experiment 5 --

| Response | F | FSQ | X1 | X2 | X4 | $X2^2$ | X1X2 | X1X4 | X2X3 | X3X4 |
|---|---|---|---|---|---|---|---|---|---|---|
| Y1 | -3.6 | | | -2.0 | | | | | | |
| Y2 | -9.2 | -4.8 | | | | | 0.7 | 0.7 | 0.7 | 0.7 |
| Y3 | 6.3 | | 10.8 | 34.0 | -11.1 | -6.2 | | | | |
| Y4 | | | 11.8 | 33.5 | -11.7 | | | | | |

-- Experiment 6 --

| Response | FSQ | X2 | X3 | X4 | X5 | X6 | X2X3 | X5X6 |
|---|---|---|---|---|---|---|---|---|
| Y1, Y4 | -17.3 | 14.4 | 11.7 | -7.0 | -13.7 | 15.7 | 4.4 | -4.0 |

-- Experiment 7 --

| Response | FSQ | X1 | X2 | X4 |
|---|---|---|---|---|
| Y2, Y4 | 15.6 | -18.0 | 15.9 | 19.3 |

-- Experiment 8 --

| Response | F | FSQ | X1 | X2 | X3 | X4 | X5 | X6 | $X1^2$ | $X4^2$ | X1X2 | X1X3 | X2X3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y1 | -2.8 | | -1.4 | 14.0 | 15.1 | | | | | | 0.9 | -3.1 | 2.2 |
| Y2 | -6.4 | -7.3 | -1.0 | | | -13.7 | 11.6 | 12.0 | | 1.2 | | | |
| Y3 | 10.4 | | 32.1 | | | | | | -5.1 | | | | |
| Y4 | | -9.8 | 30.8 | 13.4 | 13.8 | -13.4 | 11.4 | 12.0 | -5.0 | | 7.1 | | |

## Partial SS Percentage of Total SS

-- Experiment 5 --

| Response | X1 | X2 | X3 | X4 |
|---|---|---|---|---|
| Y1 | 15.1 | 50.6* | 11.2 | 23.0 |
| Y2 | 23.9* | 24.3* | 28.7* | 23.1* |
| Y3 | 9.7* | 79.3* | 1.7 | 9.3* |
| Y4 | 10.4* | 78.0* | 1.7 | 9.9* |

-- Experiment 6 --

| Response | X1 | X2 | X3 | X4 | X5 | X6 |
|---|---|---|---|---|---|---|
| Y1, Y4 | 0.4 | 24.6* | 16.8* | 6.6* | 23.1* | 28.5* |

-- Experiment 7 --

| Response | X1 | X2 | X3 | X4 |
|---|---|---|---|---|
| Y2, Y4 | 31.7* | 24.8* | 4.6 | 38.9* |

-- Experiment 8 --

| Response | X1 | X2 | X3 | X4 | X5 | X6 |
|---|---|---|---|---|---|---|
| Y1 | 2.3* | 44.5* | 53.2* | 0.0 | 0.0 | 0.0 |
| Y2 | 0.3 | 0.2 | 0.2 | 40.0* | 28.4* | 30.9* |
| Y3 | 97.0* | 1.4 | 1.4 | 0.1 | 0.1 | 0.1 |
| Y4 | 52.7* | 11.5* | 10.6* | 9.8* | 7.3* | 8.1* |

*Significant at 5% level.

factors for the combined experiment. (See Table 3a for experiment 8 factor variable definitions.) The order of importance based on the linear relative sensitivity measures from experiments 5-7 is X1, X6, X4, X5, X3, and X2. Table 3c shows that for yield (Y4), X1 is, as expected, the most influential but the order of the remaining factors is X2, X3, X4, X6, X5. Since from the results of experiments 5-7, X2 and X3 did not look as important as X4, X5, and X6, it is likely that factor X5 in Experiment 6 if selected, would have surpassed at least some of the factors in the combined experiment. This tends to show that the linear relative sensitivity measures do not provide a measure which can be compared among experiments with total confidence.

While the relative ordering of factors in experiment 8 is of interest, the main point is that all the factors have significant effects on the yield responses. Thus, the functions corresponding to all the factors should be scrutinized by the model developer with the factor order indicating where the largest improvements in yield accuracy are likely to be realized. In Figure 1, the question to be asked is how accurate are the 2 and 41°C limits on leaf appearance? From Appendix G, Table G1, it can be seen that a +5°C shift in the temperature range for which leaves can appear causes a +22% change in grain yield per hectare and a -5°C shift causes a -56% change in the yield. In Figure 2, the model developer should question the accuracy of the y-intercepts (values not given) and the shape of the family of curves. In Figure 3, the amplitude of the rate function should be examined. In Figure 4, the 5 and 40°C limits for grain filling and the height and shape of the duration curve should be reviewed. In Figure 5, the main question is how accurate is the 1.5 mg/day maximum grain filling rate?

Figure 1 - Rates of leaf initiation and appearance as a function of temperature. Leaf appearance function used in experiment 8, factor X1.

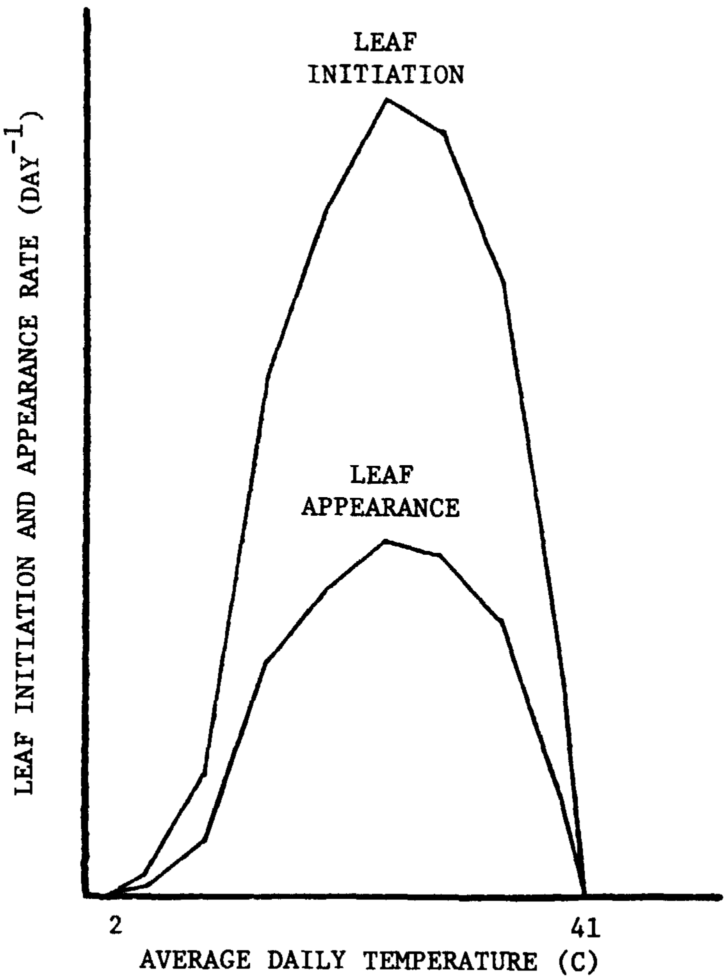


Figure 2 - Duration of the period from floral initiation to terminal spikelet initiation as a function of photoperiod and temperature used in experiment 8, factor X2.

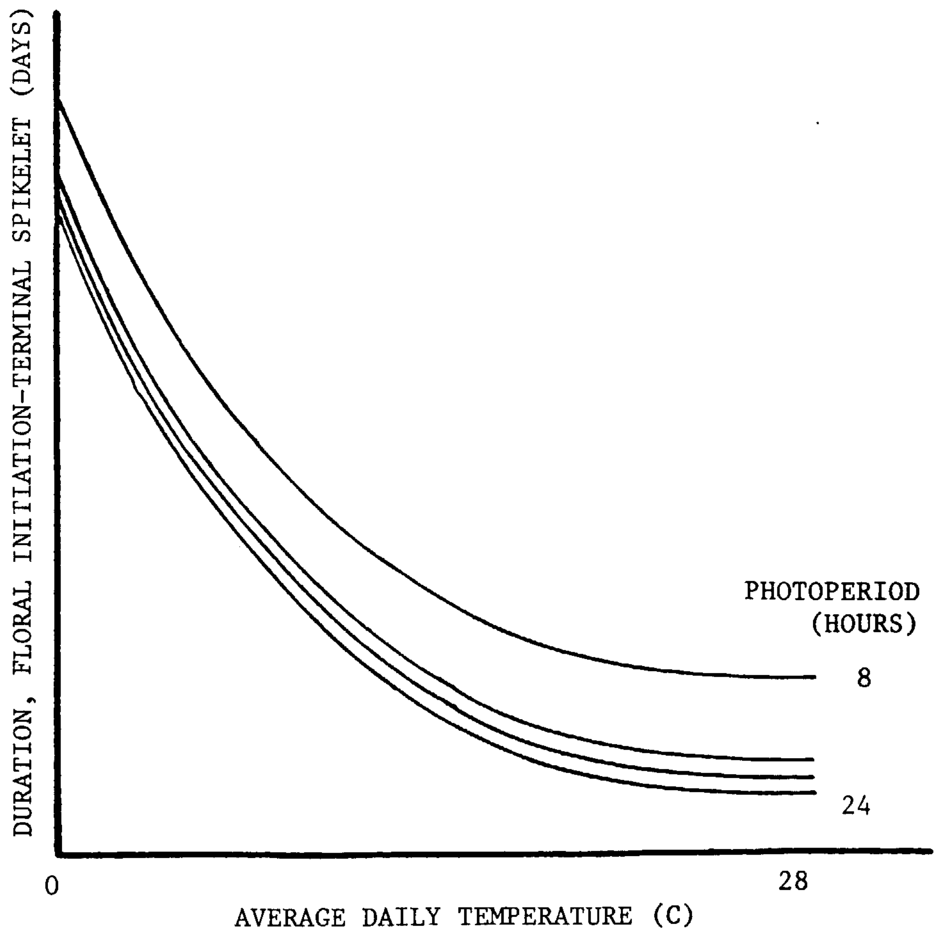


Figure 3 - Rate of floret initiation as a function of temperature used in experiment 8, factor X3.

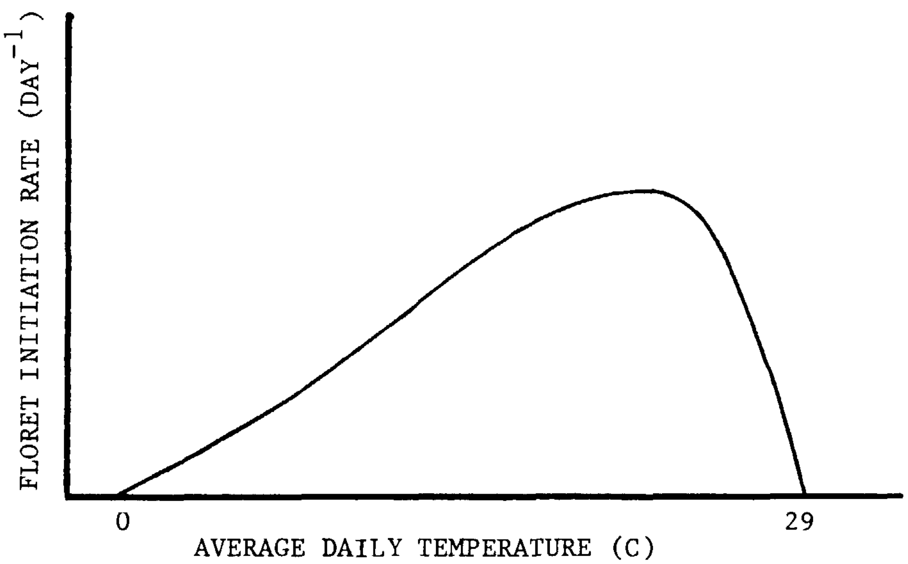


Figure 4 - Duration of the period from anthesis to physiological maturity as a function of temperature used in experiment 8, factors X4 and X5.

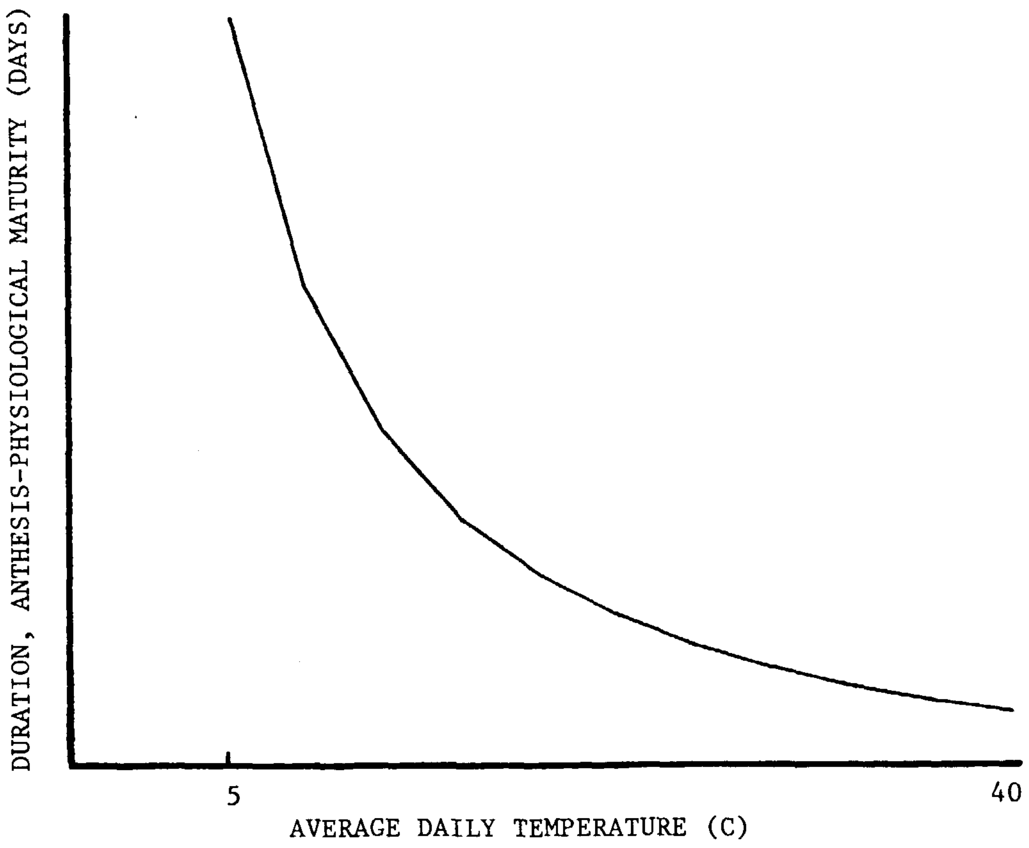
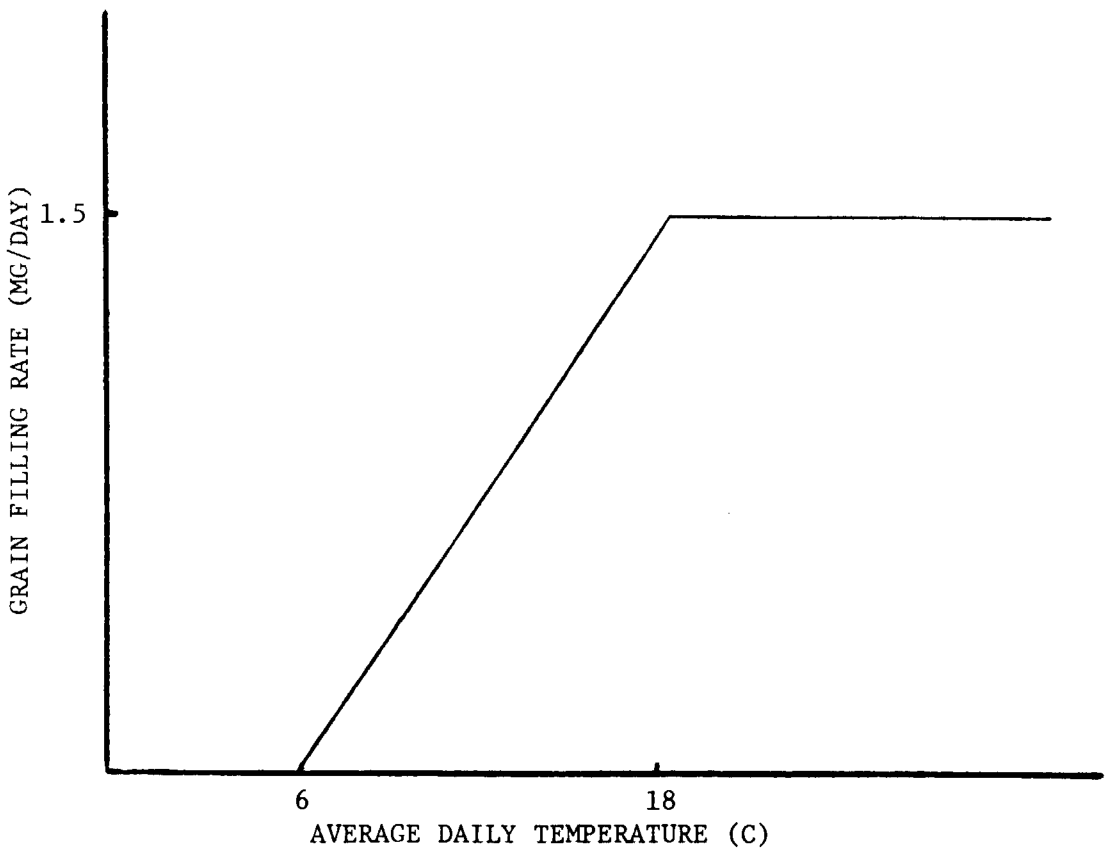


Figure 5 - Rate of grain filling as a function of temperature used in experiment 8, factor X6.



SUMMARY

The Statistical Reporting Service is interested in improving early season large-area yield forecasts. Several plant simulation models are being evaluated for large-area forecasting because plant models utilize current season environmental conditions that are not accounted for in the present operational forecasting methodology. It is hoped that plant model yield forecasts will cause a substantial improvement in forecasting accuracy in atypical growing seasons.

A two-step sensitivity analysis has been performed on the winter wheat simulation model TAMW (12) with a range of Manhattan, Kansas growing conditions. In the first step, potentially important factors were screened to determine a smaller, more manageable set. Factor screening was done with path coefficient analysis and several response surface experiments were recommended (9). This report discusses the second step in the sensitivity analysis which is to estimate model response to various factor combinations using fitted surfaces. Relative factor sensitivities were quantified using two different measures computed from the fitted surfaces.

Two series of response surface experiments were done -- one for model input variables and the other for fixed model parameters and functions. Sensitivity analysis of the input variables primarily provides insight into data needs for large-area application. Sensitivity information on parameters and functions is of most benefit to the model developer for model improvement purposes. All the response surface experiments were conducted with regard to grain yield and its components.

Results of the sensitivity analysis on TAMW input variables showed that if the initial plant available soil water is at least 60% and the seasonal rainfall is at least 11.7 inches then these water-related variables have small influence on the model yield. In Manhattan, Kansas the average rainfall for a 260-day growing season is 19.1 inches. The relationship between initial soil water and daily rainfall is such that if either exceeds the above levels then the other can be correspondingly lower without changing the model yield very much. However, if both of the water-related variables are at levels below 60% and 11.7 inches then the negative effect on yield is compounded -- 1% decreases in soil water and rainfall cause roughly a 2.6% reduction in model yield. When seasonal rainfall is above average in Manhattan, maximum and minimum temperature are by far the most critical inputs. The temperature effect on yield increases as the seasonal rainfall increases. Depending on the rainfall, a 1°F increase in temperature causes roughly a 10% increase in model yield. The temperature effect was usually linear in the sensitivity analysis but a significant

quadratic effect occurred for one climatic scenario which had a different within season temperature distribution than the other scenarios. Solar radiation has virtually no effect on the model yield within reasonable perturbation ranges.

These sensitivity analysis results are helpful for large-area application of TAMW because they provide guidelines on model input accuracy requirements. In general, systematic errors (bias) should be avoided whenever possible but random errors at the field level of, say, 10-15% on the water-related variables and, say, 4-7°F on the maximum and minimum temperature should not be of much concern over a large number of fields for most growing seasons. If growing conditions are extremely dry then the initial soil water and daily rainfall inputs may have to be estimated with less than 10% error at the field level. Solar radiation inputs can be quite imprecise.

Results of the sensitivity analysis on TAMW parameters and functions indicated several influential functions which need to be closely examined by the model developer. By far the most critical factor is the temperature range within which leaves appear on the plant. Other important functions determine the duration of spikelet initiation, the rate of floret initiation, and the rate and duration of grain filling. Concentration on improving these aspects of the model should bring about the largest gain in yield prediction accuracy.

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# APPENDIX A

Derivation of $a_o$, $F_o$, and $\alpha$ to obtain orthogonal CCD's which block orthogonally

To obtain a CCD which is orthogonal, the axial points must be located at a distance $\alpha$ which is chosen in such a way that X'X is diagonal. In general, the only off-diagonal elements that are not zero are those associated with the pure quadratic terms. According to Myers (16), the appropriate choice for $\alpha$ that makes the X'X matrix diagonal is

$$\alpha = \left(\frac{QF}{4}\right)^{1/4} \qquad [1]$$

where $Q = ((F + T)^{1/2} - F^{1/2})^2$

$F = 2^k$ = number of factorial points

$T = 2k + a_0$ = number of axial and center points

$a_0$ = number of center points

This equation for $\alpha$ was derived using a second-order model in which the pure quadratic terms had been corrected for their means. When [1] is used to compute $\alpha$ with an uncorrected model, the results are identical except for the intercept term. The difference in the intercepts is found to be

$$\hat{\beta}_0' - \hat{\beta}_0 = c \sum_{j=1}^{k} \hat{\beta}_{jj}$$

where $\hat{\beta}_0'$ = intercept in corrected model = response mean

$\hat{\beta}_0$ = intercept in uncorrected model

$\hat{\beta}_{jj}$ = $j^{th}$ pure quadratic coefficient

$c$ = correction for the pure quadratic terms

$$= \frac{F + 2\alpha^2}{F + T}$$

To divide the CCD into blocks, the axial portion forms one block and the factorial portion one or more blocks. The factorial portion is divided into blocks that form orthogonal first-order designs in the same manner that fractional replications of $2^k$ factorial designs are determined. Orthogonality within individual blocks is necessary to insure that no two main effects are aliased with each other. The number of blocks that can be formed in a CCD depends on the number of factors.

It is desirable that the block effect be orthogonal to the coefficients so that the partial sum of squares for blocks is independent of those for the coefficients. Orthogonal blocking in a CCD is obtained by appropriate selection of $\alpha$ and the number and distribution of center points among the blocks. Myers (16) states that CCD's which block orthogonally must satisfy the following condition.

$$\alpha = \left(\frac{F(2k + a_o)}{2(F + F_o)}\right)^{1/2} \qquad [2]$$

where $F_o$ = number of center points in the factorial portion

$a_o$ = number of center points in the axial portion

We wish to have an orthogonal CCD which blocks orthogonally so by equating [1] and [2], the following relationship is obtained.

$$\left(\frac{QF}{4}\right)^{1/4} = \left(\frac{FT}{2(F + F_o)}\right)^{1/2} \qquad [3]$$

where $Q = ((F + T')^{1/2} - F^{1/2})^2$

$T' = T + F_o$

$T = 2k + a_o$

$F_o$ = number of factorial center points

$a_o$ = number of axial center points

The positive solution of [3] for $F_o$ follows.

$$\left(\frac{((F + T')^{1/2} - F^{1/2})^2 F)}{4}\right)^{1/4} = \left(\frac{FT}{2(F + F_o)}\right)^{1/2}$$

$$((F + T')^{1/2} - F^{1/2})\ F^{1/2} = \frac{FT}{(F + F_o)}$$

$$(F^2 + T'F)^{1/2} = F + \frac{FT}{(F + F_o)}$$

$$(F^2 + FT + FF_o)^{1/2} = \frac{F\ (F + F_o) + FT}{(F + F_o)}$$

$$F(F + F_o) + FT = \frac{(F(F + F_o) + FT)^2}{(F + F_o)^2}$$

$$(F + F_o)^2 = F(F + F_o) + FT$$

$$(F + F_o)^2 - F(F + F_o) - FT = 0$$

$$F_o(F + F_o) - FT = 0$$

$$F_o^2 + FF_o - FT = 0$$

$$F_o = \frac{(F^2 + 4FT)^{1/2} - F}{2}$$

Thus, $F_o = (2^{2(k-1)} + 2^k(2k + a_o))^{1/2} - 2^{k-1}$ [4]

From [4] it can be seen that the number of center points and, hence, the value of $\alpha$ is not unique for a given value of k. An example for k = 3 follows. Table A1 shows the calculated $F_o$ values for several $a_o$ values using [4]. The $2^3$ factorial

Table A1 - Calculated $F_o$ values for k = 3 using [4].

| $a_o$ | $F_o$ |
|---|---|
| 0 | 4.00 |
| 1 | 4.49 |
| 2 | 4.94 |
| 3 | 5.38 |
| 4 | 5.80 |
| 5 | 6.20 |

portion can be divided into two orthogonal first-order designs each with four points. Thus $F_o$ should be an even multiple of two. For $a_o = 0$, $F_o = 4$ giving two center points in each factorial block and zero center points in the axial block. The next best choice from Table A1 would be $F_o = 6$ and $a_o = 4$ or $a_o = 5$ as both give calculated $F_o$ values equally close to six. With, say, $a_o = 4$ and $F_o = 6$, a value of $\alpha$ cannot be determined that exactly satisfies both [1] and [2]. Hence, the first choice for $a_o$ and $F_o$ would probably be preferable because exact orthogonality can be obtained. However, the second choice might be of some value in a stochastic application where an estimate for pure error and test for lack of fit are desired in the axial block.

Table A2 gives the best choice of center points for several k-values, two factorial blocks, and a deterministic application where lack of fit tests are not possible even with multiple center points. When the computed $\alpha$-values from equations [1] and [2] differ, the average provides near orthogonality.

Table A2 - Center and axial points needed to obtain an orthogonal CCD with three orthogonal blocks.

| k | 3 | 4 | 5 | 6(1/2 rep) |
|---|---|---|---|---|
| F: number of factorial points | 8 | 16 | 32 | 32 |
| $F_o$: number of added factorial center points | 4 | 6 | 8 | 10 |
| 2k: number of axial points | 6 | 8 | 10 | 12 |
| $a_o$: number of added axial center points | 0 | 0 | 0 | 1 |
| $\alpha$ using [1] | 1.414 | 1.719 | 2.000 | 2.231 |
| $\alpha$ using [2] | 1.414 | 1.706 | 2.000 | 2.225 |

# APPENDIX B

## Data and detailed analysis for Experiment 0

Table B1 - Treatment combinations and response data for experiment 0.

| OBS | F | FSQ | X1 | X2 | X3 | X4 | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | -1.000 | -1.000 | -1.000 | -1.000 | 19.29 | 18.83 | 2.54 | 1752 |
| 2 | 1 | 1 | 1.000 | 1.000 | -1.000 | -1.000 | 21.48 | 31.98 | 2.54 | 3310 |
| 3 | 1 | 1 | 1.000 | -1.000 | 1.000 | -1.000 | 21.49 | 32.21 | 2.54 | 3331 |
| 4 | 1 | 1 | 1.000 | -1.000 | -1.000 | 1.000 | 21.48 | 31.39 | 2.54 | 3247 |
| 5 | 1 | 1 | -1.000 | 1.000 | 1.000 | -1.000 | 21.49 | 33.12 | 2.54 | 3420 |
| 6 | 1 | 1 | -1.000 | 1.000 | -1.000 | 1.000 | 21.48 | 31.98 | 2.54 | 3310 |
| 7 | 1 | 1 | -1.000 | -1.000 | 1.000 | 1.000 | 18.82 | 16.89 | 2.54 | 1540 |
| 8 | 1 | 1 | 1.000 | 1.000 | 1.000 | 1.000 | 21.49 | 33.12 | 2.54 | 3420 |
| 9 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 10 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 11 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 12 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 13 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 14 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 15 | 2 | 0 | 1.000 | -1.000 | -1.000 | -1.000 | 19.52 | 32.93 | 2.79 | 3393 |
| 16 | 2 | 0 | -1.000 | 1.000 | -1.000 | -1.000 | 19.75 | 32.82 | 2.54 | 3116 |
| 17 | 2 | 0 | -1.000 | -1.000 | 1.000 | -1.000 | 21.27 | 32.17 | 3.15 | 4093 |
| 18 | 2 | 0 | -1.000 | -1.000 | -1.000 | 1.000 | 19.52 | 32.93 | 2.79 | 3393 |
| 19 | 2 | 0 | 1.000 | 1.000 | 1.000 | -1.000 | 21.27 | 32.17 | 3.15 | 4093 |
| 20 | 2 | 0 | 1.000 | 1.000 | -1.000 | 1.000 | 19.75 | 32.82 | 2.54 | 3116 |
| 21 | 2 | 0 | 1.000 | -1.000 | 1.000 | 1.000 | 21.27 | 32.17 | 3.15 | 4093 |
| 22 | 2 | 0 | -1.000 | 1.000 | 1.000 | 1.000 | 21.27 | 32.17 | 3.15 | 4093 |
| 23 | 3 | 1 | -1.712 | 0.000 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |
| 24 | 3 | 1 | 1.712 | 0.000 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |
| 25 | 3 | 1 | 0.000 | -1.712 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |
| 26 | 3 | 1 | 0.000 | 1.712 | 0.000 | 0.000 | 20.05 | 28.23 | 3.15 | 3386 |
| 27 | 3 | 1 | 0.000 | 0.000 | -1.712 | 0.000 | 15.47 | 30.54 | 2.79 | 2501 |
| 28 | 3 | 1 | 0.000 | 0.000 | 1.712 | 0.000 | 21.85 | 28.12 | 3.28 | 3818 |
| 29 | 3 | 1 | 0.000 | 0.000 | 0.000 | -1.712 | 20.06 | 28.23 | 3.15 | 3388 |
| 30 | 3 | 1 | 0.000 | 0.000 | 0.000 | 1.712 | 20.06 | 28.23 | 3.15 | 3388 |

Table B2 - Estimated second-order response surface for experiment 0, variable Y1.

| | |
|---|---|
| RESPONSE MEAN | 20.4310 |
| ROOT MSE | 1.1314 |
| R-SQUARE | 0.63928278 |
| COEF OF VARIATION | 0.05537638 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 7.8422 | 0.1700 | 3.06 | 0.0058 |
| LINEAR | 4 | 15.6317 | 0.3388 | 3.05 | 0.0560 |
| QUADRATIC | 4 | 2.9811 | 0.0646 | 0.58 | 0.6810 |
| CROSSPRODUCT | 6 | 3.0365 | 0.0658 | 0.40 | 0.8691 |
| TOTAL REGRESS | 16 | 29.4916 | 0.6393 | 1.44 | 0.2565 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 16.6407 | 1.8490 | 669078.320 | 0.0001 |
| PURE ERROR 1) | 4 | 1.70530E-13 | 4.26326E-14 | | |
| TOTAL ERROR | 13 | 16.6407 | 1.2801 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 21.7543 | 0.73863363 | 29.45 | 0.0001 |
| X1 | 1 | 0.22230468 | 0.24197491 | 0.92 | 0.3750 |
| X2 | 1 | 0.24256276 | 0.24197491 | 1.00 | 0.3344 |
| X3 | 1 | 0.77864089 | 0.24197491 | 3.22 | 0.0067 |
| X4 | 1 | -0.02195602 | 0.24197491 | -0.09 | 0.9291 |
| X1*X1 | 1 | 0.09359193 | 0.27241987 | 0.34 | 0.7367 |
| X1*X2 | 1 | -0.30375000 | 0.28284870 | -1.07 | 0.3024 |
| X2*X2 | 1 | 0.09188600 | 0.27241987 | 0.34 | 0.7413 |
| X1*X3 | 1 | 0.03000000 | 0.28284870 | 0.11 | 0.9172 |
| X2*X3 | 1 | 0.00125000 | 0.28284870 | 0.00 | 0.9965 |
| X3*X3 | 1 | -0.38406987 | 0.27241987 | -1.41 | 0.1821 |
| X1*X4 | 1 | 0.05875000 | 0.28284870 | 0.21 | 0.8387 |
| X2*X4 | 1 | 0.03000000 | 0.28284870 | 0.11 | 0.9172 |
| X3*X4 | 1 | -0.30375000 | 0.28284870 | -1.07 | 0.3024 |
| X4*X4 | 1 | 0.09359193 | 0.27241987 | 0.34 | 0.7367 |
| F | 1 | -0.64852000 | 0.26286122 | -2.47 | 0.0283 |
| FSQ | 1 | -0.02306545 | 0.43065599 | -0.05 | 0.9581 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 2.7773 | 0.55546758 | 0.43 | 0.8171 |
| X2 | 5 | 2.9226 | 0.58451215 | 0.46 | 0.8014 |
| X3 | 5 | 17.2894 | 3.4579 | 2.70 | 0.0691 |
| X4 | 5 | 1.7075 | 0.34149521 | 0.27 | 0.9233 |

1) Since TAMW is a deterministic model, the pure error is zero. Hence, the LOF mean square error should be used for all hypotheses tests rather than the total MSE as has been done here.

Table B3 - Estimated second-order response surface for experiment 0, variable Y2.

| | |
|---|---|
| RESPONSE MEAN | 30.6077 |
| ROOT MSE | 3.6287 |
| R-SQUARE | 0.63366176 |
| COEF OF VARIATION | 0.11855506 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 79.8394 | 0.1709 | 3.03 | 0.0001 |
| LINEAR | 4 | 80.4424 | 0.1722 | 1.53 | 0.2517 |
| QUADRATIC | 4 | 36.2401 | 0.0776 | 0.69 | 0.6130 |
| CROSSPRODUCT | 6 | 99.5650 | 0.2131 | 1.26 | 0.3396 |
| TOTAL REGRESS | 16 | 296.0869 | 0.6337 | 1.41 | 0.2710 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 171.1764 | 19.0196 | 347324.810 | 0.0001 |
| PURE ERROR 1) | 4 | 2.27374E-13 | 5.68434E-14 | | |
| TOTAL ERROR | 13 | 171.1764 | 13.1674 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 36.2669 | 2.3690 | 15.31 | 0.0001 |
| X1 | 1 | 1.2753 | 0.77607996 | 1.64 | 0.1243 |
| X2 | 1 | 1.4024 | 0.77607996 | 1.81 | 0.0939 |
| X3 | 1 | -0.26544094 | 0.77607996 | -0.34 | 0.7378 |
| X4 | 1 | -0.12624710 | 0.77607996 | -0.16 | 0.8733 |
| X1*X1 | 1 | -0.79601393 | 0.87372531 | -0.91 | 0.3788 |
| X1*X2 | 1 | -1.7425 | 0.90717341 | -1.92 | 0.0770 |
| X2*X2 | 1 | -0.79601393 | 0.87372531 | -0.91 | 0.3788 |
| X1*X3 | 1 | 0.17250000 | 0.90717341 | 0.19 | 0.8521 |
| X2*X3 | 1 | 0.22625000 | 0.90717341 | 0.25 | 0.8069 |
| X3*X3 | 1 | -0.42070822 | 0.87372531 | -0.48 | 0.6382 |
| X1*X4 | 1 | 0.19875000 | 0.90717341 | 0.22 | 0.8300 |
| X2*X4 | 1 | 0.17250000 | 0.90717341 | 0.19 | 0.8521 |
| X3*X4 | 1 | -1.7425 | 0.90717341 | -1.92 | 0.0770 |
| X4*X4 | 1 | -0.79601393 | 0.87372531 | -0.91 | 0.3788 |
| F | 1 | -0.82664570 | 0.84306809 | -0.98 | 0.3447 |
| FSQ | 1 | -3.2239 | 1.3812 | -2.33 | 0.0363 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 96.1731 | 19.2346 | 1.46 | 0.2681 |
| X2 | 5 | 103.8041 | 20.7608 | 1.58 | 0.2347 |
| X3 | 5 | 54.4693 | 10.8939 | 0.83 | 0.5523 |
| X4 | 5 | 60.9668 | 12.1934 | 0.93 | 0.4951 |

1) See footnote at bottom of Table B2.

Table B4 - Estimated second-order response surface for experiment 0, variable Y3.

| | |
|---|---|
| RESPONSE MEAN | 2.8540 |
| ROOT MSE | 0.15719695 |
| R-SQUARE | 0.87713177 |
| COEF OF VARIATION | 0.05507952 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 1.8134 | 0.6936 | 36.69 | 0.2020 |
| LINEAR | 4 | 0.36466082 | 0.1395 | 3.69 | 0.0322 |
| QUADRATIC | 4 | 0.08398977 | 0.0321 | 0.85 | 0.5187 |
| CROSSPRODUCT | 6 | 0.03125000 | 0.0120 | 0.21 | 0.9669 |
| TOTAL REGRESS | 16 | 2.2933 | 0.8771 | 5.80 | 0.0014 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 0.32124145 | 0.03569349 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 4 | 0 | 0 | | |
| TOTAL ERROR | 13 | 0.32124145 | 0.02471088 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 2.4799 | 0.10262638 | 24.16 | 0.0001 |
| X1 | 1 | 0 | 0.03362020 | 0.00 | 1.0000 |
| X2 | 1 | -0.02287085 | 0.03362020 | -0.68 | 0.5083 |
| X3 | 1 | 0.12711071 | 0.03362020 | 3.78 | 0.0023 |
| X4 | 1 | 0 | 0.03362020 | 0.00 | 1.0000 |
| X1*X1 | 1 | -0.02050337 | 0.03785025 | -0.54 | 0.5972 |
| X1*X2 | 1 | 0 | 0.03929924 | 0.00 | 1.0000 |
| X2*X2 | 1 | -0.02050337 | 0.03785025 | -0.54 | 0.5972 |
| X1*X3 | 1 | 0 | 0.03929924 | 0.00 | 1.0000 |
| X2*X3 | 1 | 0.03125000 | 0.03929924 | 0.80 | 0.4408 |
| X3*X3 | 1 | -0.05973988 | 0.03785025 | -1.58 | 0.1385 |
| X1*X4 | 1 | -0.03125000 | 0.03929924 | -0.80 | 0.4408 |
| X2*X4 | 1 | 0 | 0.03929924 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.03929924 | 0.00 | 1.0000 |
| X4*X4 | 1 | -0.02050337 | 0.03785025 | -0.54 | 0.5972 |
| F | 1 | 0.29095621 | 0.03652216 | 7.97 | 0.0001 |
| FSQ | 1 | -0.14268015 | 0.05983571 | -2.38 | 0.0330 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.02287606 | 0.00457521 | 0.19 | 0.9632 |
| X2 | 5 | 0.03431148 | 0.00686230 | 0.28 | 0.9172 |
| X3 | 5 | 0.43040768 | 0.08608154 | 3.48 | 0.0323 |
| X4 | 5 | 0.02287606 | 0.00457521 | 0.19 | 0.9632 |

1) See footnote at bottom of Table B2.

Table B5 - Estimated second-order response surface for experiment 0, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 3390.6000 |
| ROOT MSE | 480.1561 |
| R-SQUARE | 0.70604762 |
| COEF OF VARIATION | 0.14161390 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 2492804 | 0.2445 | 5.41 | 0.0001 |
| LINEAR | 4 | 2405097 | 0.2359 | 2.61 | 0.0846 |
| QUADRATIC | 4 | 886805 | 0.0870 | 0.96 | 0.4608 |
| CROSSPRODUCT | 6 | 1414179 | 0.1387 | 1.02 | 0.4531 |
| TOTAL REGRESS | 16 | 7198885 | 0.7060 | 1.95 | 0.1148 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 2997148 | 333016 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 4 | 0 | 0 | | |
| TOTAL ERROR | 13 | 2997148 | 230550 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3804.9322 | 313.4710 | 12.14 | 0.0001 |
| X1 | 1 | 150.3072 | 102.6925 | 1.46 | 0.1670 |
| X2 | 1 | 138.7152 | 102.6925 | 1.35 | 0.1998 |
| X3 | 1 | 260.7599 | 102.6925 | 2.54 | 0.0247 |
| X4 | 1 | -13.5395 | 102.6925 | -0.13 | 0.8971 |
| X1*X1 | 1 | -88.0494 | 115.6131 | -0.76 | 0.4599 |
| X1*X2 | 1 | -205.3750 | 120.0390 | -1.71 | 0.1108 |
| X2*X2 | 1 | -88.3906 | 115.6131 | -0.76 | 0.4582 |
| X1*X3 | 1 | 18.5000 | 120.0390 | 0.15 | 0.8799 |
| X2*X3 | 1 | 56.3750 | 120.0390 | 0.47 | 0.6464 |
| X3*X3 | 1 | -166.0106 | 115.6131 | -1.44 | 0.1747 |
| X1*X4 | 1 | -12.8750 | 120.0390 | -0.11 | 0.9162 |
| X2*X4 | 1 | 18.5000 | 120.0390 | 0.15 | 0.8799 |
| X3*X4 | 1 | -205.3750 | 120.0390 | -1.71 | 0.1108 |
| X4*X4 | 1 | -88.0494 | 115.6131 | -0.76 | 0.4599 |
| F | 1 | 127.2612 | 111.5565 | 1.14 | 0.2745 |
| FSQ | 1 | -540.6479 | 182.7674 | -2.96 | 0.0111 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 1310622 | 262124 | 1.14 | 0.3894 |
| X2 | 5 | 1286613 | 257323 | 1.12 | 0.3989 |
| X3 | 5 | 2693063 | 538613 | 2.34 | 0.1011 |
| X4 | 5 | 820720 | 164144 | 0.71 | 0.6253 |

1) See footnote at bottom of Table B2.

# APPENDIX C

## Data and detailed analysis for Experiment 1

Table C1 - Treatment combinations and response data for experiment 1.

| OBS | X1 | X2 | X3 | X4 | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|---|---|---|---|
| 1 | -1.000 | -1.000 | -1.000 | -1.000 | 17.03 | 16.11 | 2.54 | 1331 |
| 2 | -1.000 | -1.000 | -1.000 | 1.000 | 15.67 | 14.53 | 2.54 | 1100 |
| 3 | -1.000 | -1.000 | 1.000 | -1.000 | 16.65 | 15.85 | 2.54 | 1281 |
| 4 | -1.000 | -1.000 | 1.000 | 1.000 | 14.99 | 13.78 | 2.54 | 1002 |
| 5 | -1.000 | 1.000 | -1.000 | -1.000 | 21.53 | 31.70 | 2.54 | 3288 |
| 6 | -1.000 | 1.000 | -1.000 | 1.000 | 21.53 | 31.70 | 2.54 | 3288 |
| 7 | -1.000 | 1.000 | 1.000 | -1.000 | 21.55 | 33.14 | 2.54 | 3433 |
| 8 | -1.000 | 1.000 | 1.000 | 1.000 | 21.55 | 31.64 | 2.54 | 3278 |
| 9 | 1.000 | -1.000 | -1.000 | -1.000 | 21.53 | 31.70 | 2.54 | 3288 |
| 10 | 1.000 | -1.000 | -1.000 | 1.000 | 21.53 | 31.70 | 2.54 | 3288 |
| 11 | 1.000 | -1.000 | 1.000 | -1.000 | 21.55 | 31.64 | 2.54 | 3278 |
| 12 | 1.000 | -1.000 | 1.000 | 1.000 | 21.55 | 30.14 | 2.54 | 3122 |
| 13 | 1.000 | 1.000 | -1.000 | -1.000 | 21.53 | 31.70 | 2.54 | 3288 |
| 14 | 1.000 | 1.000 | -1.000 | 1.000 | 21.53 | 31.70 | 2.54 | 3288 |
| 15 | 1.000 | 1.000 | 1.000 | -1.000 | 21.55 | 33.14 | 2.54 | 3433 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 | 21.55 | 33.14 | 2.54 | 3433 |
| 17 | -1.414 | 0.000 | 0.000 | 0.000 | 19.53 | 21.87 | 2.54 | 2065 |
| 18 | 1.414 | 0.000 | 0.000 | 0.000 | 21.35 | 34.45 | 2.54 | 3542 |
| 19 | 0.000 | -1.414 | 0.000 | 0.000 | 20.25 | 18.75 | 2.54 | 1837 |
| 20 | 0.000 | 1.414 | 0.000 | 0.000 | 21.35 | 34.45 | 2.54 | 3542 |
| 21 | 0.000 | 0.000 | -1.414 | 0.000 | 22.12 | 30.70 | 2.54 | 3272 |
| 22 | 0.000 | 0.000 | 1.414 | 0.000 | 21.60 | 31.45 | 2.79 | 3585 |
| 23 | 0.000 | 0.000 | 0.000 | -1.414 | 21.35 | 34.45 | 2.54 | 3542 |
| 24 | 0.000 | 0.000 | 0.000 | 1.414 | 21.35 | 33.54 | 2.54 | 3455 |
| 25 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |

Table C2 - Estimated second-order response surface for experiment 1, variable Y1.

| | |
|---|---|
| RESPONSE MEAN | 20.5228 |
| ROOT MSE | 0.76406052 |
| R-SQUARE | 0.94274906 |
| COEF OF VARIATION | 0.03722984 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 57.6724 | 0.5656 | 24.70 | 0.0001 |
| QUADRATIC | 4 | 7.4059 | 0.0726 | 3.17 | 0.0632 |
| CROSSPRODUCT | 6 | 31.0539 | 0.3045 | 8.87 | 0.0016 |
| TOTAL REGRESS | 14 | 96.1322 | 0.9427 | 11.76 | 0.0002 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 5.8379 | 0.58378848 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 21.7969 | 0.45833787 | 47.56 | 0.0001 |
| X1 | 1 | 1.2197 | 0.17085429 | 7.14 | 0.0001 |
| X2 | 1 | 1.1688 | 0.17085429 | 6.84 | 0.0001 |
| X3 | 1 | -0.08376906 | 0.17085429 | -0.49 | 0.6345 |
| X4 | 1 | -0.15100912 | 0.17085429 | -0.88 | 0.3975 |
| X1*X1 | 1 | -0.73454629 | 0.27018516 | -2.72 | 0.0216 |
| X1*X2 | 1 | -1.3637 | 0.19101513 | -7.14 | 0.0001 |
| X2*X2 | 1 | -0.55449192 | 0.27018516 | -2.05 | 0.0673 |
| X1*X3 | 1 | 0.06875000 | 0.19101513 | 0.36 | 0.7264 |
| X2*X3 | 1 | 0.06875000 | 0.19101513 | 0.36 | 0.7264 |
| X3*X3 | 1 | -0.02433181 | 0.27018516 | -0.09 | 0.9300 |
| X1*X4 | 1 | 0.18875000 | 0.19101513 | 0.99 | 0.3464 |
| X2*X4 | 1 | 0.18875000 | 0.19101513 | 0.99 | 0.3464 |
| X3*X4 | 1 | -0.01875000 | 0.19101513 | -0.10 | 0.9237 |
| X4*X4 | 1 | -0.27940884 | 0.27018516 | -1.03 | 0.3254 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 64.4715 | 12.8943 | 22.09 | 0.0001 |
| X2 | 5 | 60.1836 | 12.0367 | 20.62 | 0.0001 |
| X3 | 5 | 0.30194621 | 0.06038924 | 0.10 | 0.9891 |
| X4 | 5 | 2.2261 | 0.44521011 | 0.76 | 0.5968 |

Table C3 - Estimated second-order response surface for experiment 1, variable Y2.

| | |
|---|---|
| RESPONSE MEAN | 28.6840 |
| ROOT MSE | 0.82832099 |
| R-SQUARE | 0.99422866 |
| COEF OF VARIATION | 0.02887746 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 805.5771 | 0.6776 | 293.53 | 0.0001 |
| QUADRATIC | 4 | 119.6499 | 0.1006 | 43.60 | 0.0001 |
| CROSSPRODUCT | 6 | 256.7448 | 0.2160 | 62.37 | 0.0001 |
| TOTAL REGRESS | 14 | 1181.9718 | 0.9942 | 123.05 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 6.8612 | 0.68611567 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 32.8988 | 0.49688588 | 66.21 | 0.0001 |
| X1 | 1 | 4.2102 | 0.18522380 | 22.73 | 0.0001 |
| X2 | 1 | 4.7308 | 0.18522380 | 25.54 | 0.0001 |
| X3 | 1 | 0.13453313 | 0.18522380 | 0.73 | 0.4843 |
| X4 | 1 | -0.39686097 | 0.18522380 | -2.14 | 0.0578 |
| X1*X1 | 1 | -2.2162 | 0.29290879 | -7.57 | 0.0001 |
| X1*X2 | 1 | -3.9631 | 0.20708025 | -19.14 | 0.0001 |
| X2*X2 | 1 | -2.9965 | 0.29290879 | -10.23 | 0.0001 |
| X1*X3 | 1 | 0.05562500 | 0.20708025 | 0.27 | 0.7937 |
| X2*X3 | 1 | 0.43062500 | 0.20708025 | 2.08 | 0.0643 |
| X3*X3 | 1 | -0.75829606 | 0.29290879 | -2.59 | 0.0270 |
| X1*X4 | 1 | 0.22812500 | 0.20708025 | 1.10 | 0.2964 |
| X2*X4 | 1 | 0.22812500 | 0.20708025 | 1.10 | 0.2964 |
| X3*X4 | 1 | -0.21812500 | 0.20708025 | -1.05 | 0.3170 |
| X4*X4 | 1 | 0.70214499 | 0.29290879 | 2.40 | 0.0375 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 645.9509 | 129.1902 | 188.29 | 0.0001 |
| X2 | 5 | 774.4839 | 154.8968 | 225.76 | 0.0001 |
| X3 | 5 | 8.7382 | 1.7476 | 2.55 | 0.0977 |
| X4 | 5 | 9.5190 | 1.9038 | 2.77 | 0.0797 |

Table C4 - Estimated second-order response surface for experiment 1, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 2910.8000 |
| ROOT MSE | 73.3943 |
| R-SQUARE | 0.99706696 |
| COEF OF VARIATION | 0.02521447 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 12133295 | 0.6607 | 563.11 | 0.0001 |
| QUADRATIC | 4 | 1994118 | 0.1086 | 92.55 | 0.0001 |
| CROSSPRODUCT | 6 | 4184354 | 0.2278 | 129.47 | 0.0001 |
| TOTAL REGRESS | 14 | 18311767 | 0.9971 | 242.82 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 53867.2210 | 5386.7221 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3461.4610 | 44.0271 | 78.62 | 0.0001 |
| X1 | 1 | 525.3056 | 16.4120 | 32.01 | 0.0001 |
| X2 | 1 | 572.5281 | 16.4120 | 34.88 | 0.0001 |
| X3 | 1 | 27.1807 | 16.4120 | 1.66 | 0.1287 |
| X4 | 1 | -47.2038 | 16.4120 | -2.88 | 0.0165 |
| X1*X1 | 1 | -322.8875 | 25.9535 | -12.44 | 0.0001 |
| X1*X2 | 1 | -506.6875 | 18.3486 | -27.61 | 0.0001 |
| X2*X2 | 1 | -379.9047 | 25.9535 | -14.64 | 0.0001 |
| X1*X3 | 1 | 7.9375 | 18.3486 | 0.43 | 0.6745 |
| X2*X3 | 1 | 46.8125 | 18.3486 | 2.55 | 0.0288 |
| X3*X3 | 1 | -10.2931 | 25.9535 | -0.40 | 0.7000 |
| X1*X4 | 1 | 31.8125 | 18.3486 | 1.73 | 0.1136 |
| X2*X4 | 1 | 31.9375 | 18.3486 | 1.74 | 0.1124 |
| X3*X4 | 1 | -22.4375 | 18.3486 | -1.22 | 0.2494 |
| X4*X4 | 1 | 24.7175 | 25.9535 | 0.95 | 0.3634 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 10477251 | 2095450 | 389.00 | 0.0001 |
| X2 | 5 | 11868672 | 2373734 | 440.66 | 0.0001 |
| X3 | 5 | 59747.9229 | 11949.5846 | 2.22 | 0.1328 |
| X4 | 5 | 90014.7406 | 18002.9481 | 3.34 | 0.0493 |

APPENDIX D

Data and detailed analysis for Experiment 2

Table D1 - Treatment combinations and response data for experiment 2.

| OBS | X1 | X2 | X3 | X4 | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|---|---|---|---|
| 1 | -1.000 | -1.000 | -1.000 | -1.000 | 18.99 | 33.45 | 2.79 | 3357 |
| 2 | -1.000 | -1.000 | -1.000 | 1.000 | 18.99 | 33.45 | 2.79 | 3357 |
| 3 | -1.000 | -1.000 | 1.000 | -1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 4 | -1.000 | -1.000 | 1.000 | 1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 5 | -1.000 | 1.000 | -1.000 | -1.000 | 19.12 | 33.39 | 2.54 | 3072 |
| 6 | -1.000 | 1.000 | -1.000 | 1.000 | 19.12 | 33.39 | 2.54 | 3072 |
| 7 | -1.000 | 1.000 | 1.000 | -1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 8 | -1.000 | 1.000 | 1.000 | 1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 9 | 1.000 | -1.000 | -1.000 | -1.000 | 18.99 | 33.45 | 2.79 | 3357 |
| 10 | 1.000 | -1.000 | -1.000 | 1.000 | 18.99 | 33.45 | 2.79 | 3357 |
| 11 | 1.000 | -1.000 | 1.000 | -1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 12 | 1.000 | -1.000 | 1.000 | 1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 13 | 1.000 | 1.000 | -1.000 | -1.000 | 19.12 | 33.39 | 2.54 | 3072 |
| 14 | 1.000 | 1.000 | -1.000 | 1.000 | 19.12 | 33.39 | 2.54 | 3072 |
| 15 | 1.000 | 1.000 | 1.000 | -1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 | 22.50 | 33.14 | 3.15 | 4457 |
| 17 | -1.414 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 18 | 1.414 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 19 | 0.000 | -1.414 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 20 | 0.000 | 1.414 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3942 |
| 21 | 0.000 | 0.000 | -1.414 | 0.000 | 19.04 | 35.36 | 2.54 | 3240 |
| 22 | 0.000 | 0.000 | 1.414 | 0.000 | 22.72 | 32.76 | 3.15 | 4459 |
| 23 | 0.000 | 0.000 | 0.000 | -1.414 | 20.19 | 32.70 | 3.15 | 3940 |
| 24 | 0.000 | 0.000 | 0.000 | 1.414 | 20.19 | 32.70 | 3.15 | 3940 |
| 25 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |

Table D2 - Estimated second-order response surface for experiment 2, variable Y1.

| | |
|---|---|
| RESPONSE MEAN | 20.6212 |
| ROOT MSE | 0.24669665 |
| R-SQUARE | 0.98915232 |
| COEF OF VARIATION | 0.01196325 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 53.6892 | 0.9570 | 220.55 | 0.0001 |
| QUADRATIC | 4 | 1.7888 | 0.0319 | 7.35 | 0.0050 |
| CROSSPRODUCT | 6 | 0.01690000 | 0.0003 | 0.05 | 0.9994 |
| TOTAL REGRESS | 14 | 55.4949 | 0.9892 | 65.13 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 0.60859237 | 0.06085924 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 20.0349 | 0.14798621 | 135.38 | 0.0001 |
| X1 | 1 | 0 | 0.05516471 | 0.00 | 1.0000 |
| X2 | 1 | 0.02600157 | 0.05516471 | 0.47 | 0.6475 |
| X3 | 1 | 1.6383 | 0.05516471 | 29.70 | 0.0001 |
| X4 | 1 | 0 | 0.05516471 | 0.00 | 1.0000 |
| X1*X1 | 1 | 0.09694660 | 0.08723625 | 1.11 | 0.2924 |
| X1*X2 | 1 | 0 | 0.06167416 | 0.00 | 1.0000 |
| X2*X2 | 1 | 0.09694660 | 0.08723625 | 1.11 | 0.2924 |
| X1*X3 | 1 | 0 | 0.06167416 | 0.00 | 1.0000 |
| X2*X3 | 1 | -0.03250000 | 0.06167416 | -0.53 | 0.6097 |
| X3*X3 | 1 | 0.44205082 | 0.08723625 | 5.07 | 0.0005 |
| X1*X4 | 1 | 0 | 0.06167416 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.06167416 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.06167416 | 0.00 | 1.0000 |
| X4*X4 | 1 | 0.09694660 | 0.08723625 | 1.11 | 0.2924 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.07516189 | 0.01503238 | 0.25 | 0.9320 |
| X2 | 5 | 0.10558270 | 0.02111654 | 0.35 | 0.8729 |
| X3 | 5 | 55.2553 | 11.0511 | 181.58 | 0.0001 |
| X4 | 5 | 0.07516189 | 0.01503238 | 0.25 | 0.9320 |

Table D3 - Estimated second-order response surface for experiment 2, variable Y2.

| | |
|---|---|
| RESPONSE MEAN | 33.1800 |
| ROOT MSE | 0.44163840 |
| R-SQUARE | 0.72225382 |
| COEF OF VARIATION | 0.01331038 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 1.7532 | 0.2497 | 2.25 | 0.1364 |
| QUADRATIC | 4 | 3.3152 | 0.4721 | 4.25 | 0.0289 |
| CROSSPRODUCT | 6 | 0.00360000 | 0.0005 | 0.00 | 1.0000 |
| TOTAL REGRESS | 14 | 5.0720 | 0.7223 | 1.86 | 0.1640 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 1.9504 | 0.19504448 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 32.7641 | 0.26492614 | 123.67 | 0.0001 |
| X1 | 1 | 1.42117E-15 | 0.09875633 | 0.00 | 1.0000 |
| X2 | 1 | -0.01200072 | 0.09875633 | -0.12 | 0.9057 |
| X3 | 1 | -0.29583787 | 0.09875633 | -3.00 | 0.0134 |
| X4 | 1 | 0 | 0.09875633 | 0.00 | 1.0000 |
| X1*X1 | 1 | -0.04007733 | 0.15617106 | -0.26 | 0.8027 |
| X1*X2 | 1 | 0 | 0.11040960 | 0.00 | 1.0000 |
| X2*X2 | 1 | -0.04007733 | 0.15617106 | -0.26 | 0.8027 |
| X1*X3 | 1 | 0 | 0.11040960 | 0.00 | 1.0000 |
| X2*X3 | 1 | 0.01500000 | 0.11040960 | 0.14 | 0.8946 |
| X3*X3 | 1 | 0.64012810 | 0.15617106 | 4.10 | 0.0021 |
| X1*X4 | 1 | 0 | 0.11040960 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.11040960 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.11040960 | 0.00 | 1.0000 |
| X4*X4 | 1 | -0.04007733 | 0.15617106 | -0.26 | 0.8027 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.01284488 | 0.00256898 | 0.01 | 1.0000 |
| X2 | 5 | 0.01932505 | 0.00386501 | 0.02 | 0.9998 |
| X3 | 5 | 5.0308 | 1.0062 | 5.16 | 0.0134 |
| X4 | 5 | 0.01284488 | 0.00256898 | 0.01 | 1.0000 |

Table D4 - Estimated second-order response surface for experiment 2, variable Y3.

| | |
|---|---|
| RESPONSE MEAN | 2.9704 |
| ROOT MSE | 0.04468608 |
| R-SQUARE | 0.98730142 |
| COEF OF VARIATION | 0.01504379 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 1.1747 | 0.7470 | 147.06 | 0.0001 |
| QUADRATIC | 4 | 0.31537231 | 0.2006 | 39.48 | 0.0001 |
| CROSSPRODUCT | 6 | 0.06250000 | 0.0397 | 5.22 | 0.0112 |
| TOTAL REGRESS | 14 | 1.5525 | 0.9873 | 55.53 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 0.01996846 | 0.00199685 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3.2075 | 0.02680589 | 119.66 | 0.0001 |
| X1 | 1 | 5.10733E-16 | 0.00999241 | 0.00 | 1.0000 |
| X2 | 1 | -0.05000302 | 0.00999241 | -5.00 | 0.0005 |
| X3 | 1 | 0.23714132 | 0.00999241 | 23.73 | 0.0001 |
| X4 | 1 | 0 | 0.00999241 | 0.00 | 1.0000 |
| X1*X1 | 1 | -0.03597723 | 0.01580178 | -2.28 | 0.0460 |
| X1*X2 | 1 | 0 | 0.01117152 | 0.00 | 1.0000 |
| X2*X2 | 1 | -0.03597723 | 0.01580178 | -2.28 | 0.0460 |
| X1*X3 | 1 | 0 | 0.01117152 | 0.00 | 1.0000 |
| X2*X3 | 1 | 0.06250000 | 0.01117152 | 5.59 | 0.0002 |
| X3*X3 | 1 | -0.18852329 | 0.01580178 | -11.93 | 0.0001 |
| X1*X4 | 1 | 0 | 0.01117152 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.01117152 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.01117152 | 0.00 | 1.0000 |
| X4*X4 | 1 | -0.03597723 | 0.01580178 | -2.28 | 0.0460 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.01035113 | 0.00207023 | 1.04 | 0.4472 |
| X2 | 5 | 0.12285415 | 0.02457083 | 12.30 | 0.0005 |
| X3 | 5 | 1.4714 | 0.29427548 | 147.37 | 0.0001 |
| X4 | 5 | 0.01035113 | 0.00207023 | 1.04 | 0.4472 |

Table D5 - Estimated second-order response surface for experiment 2, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 3866.1200 |
| ROOT MSE | 116.0030 |
| R-SQUARE | 0.98113848 |
| COEF OF VARIATION | 0.03000503 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 6867128 | 0.9625 | 127.58 | 0.0001 |
| QUADRATIC | 4 | 51552.5972 | 0.0072 | 0.96 | 0.4712 |
| CROSSPRODUCT | 6 | 81225.0000 | 0.0114 | 1.01 | 0.4722 |
| TOTAL REGRESS | 14 | 6999906 | 0.9811 | 37.16 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 134567 | 13456.7032 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3978.0576 | 69.5869 | 57.17 | 0.0001 |
| X1 | 1 | 0 | 25.9398 | 0.00 | 1.0000 |
| X2 | 1 | -56.8620 | 25.9398 | -2.19 | 0.0531 |
| X3 | 1 | 583.2185 | 25.9398 | 22.48 | 0.0001 |
| X4 | 1 | 0 | 25.9398 | 0.00 | 1.0000 |
| X1*X1 | 1 | -23.7917 | 41.0207 | -0.58 | 0.5748 |
| X1*X2 | 1 | 0 | 29.0008 | 0.00 | 1.0000 |
| X2*X2 | 1 | -23.2916 | 41.0207 | -0.57 | 0.5827 |
| X1*X3 | 1 | 0 | 29.0008 | 0.00 | 1.0000 |
| X2*X3 | 1 | 71.2500 | 29.0008 | 2.46 | 0.0339 |
| X3*X3 | 1 | -69.0554 | 41.0207 | -1.68 | 0.1232 |
| X1*X4 | 1 | 0 | 29.0008 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 29.0008 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 29.0008 | 0.00 | 1.0000 |
| X4*X4 | 1 | -23.7917 | 41.0207 | -0.58 | 0.5748 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 4526.7252 | 905.3450 | 0.07 | 0.9959 |
| X2 | 5 | 150225 | 30045.0634 | 2.23 | 0.1310 |
| X3 | 5 | 6921826 | 1384365 | 102.88 | 0.0001 |
| X4 | 5 | 4526.7252 | 905.3450 | 0.07 | 0.9959 |

## APPENDIX E

**Data and detailed analysis for Experiment 3**

Table E1 - Treatment combinations and response data for experiment 3.

| OBS | X1 | X2 | X3 | X4 | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|---|---|---|---|
| 1 | -1.000 | -1.000 | -1.000 | -1.000 | 17.01 | 29.26 | 3.15 | 2971 |
| 2 | -1.000 | -1.000 | -1.000 | 1.000 | 17.01 | 28.79 | 3.15 | 2925 |
| 3 | -1.000 | -1.000 | 1.000 | -1.000 | 22.22 | 28.47 | 3.28 | 3936 |
| 4 | -1.000 | -1.000 | 1.000 | 1.000 | 22.22 | 27.10 | 3.28 | 3744 |
| 5 | -1.000 | 1.000 | -1.000 | -1.000 | 17.04 | 29.81 | 3.15 | 3025 |
| 6 | -1.000 | 1.000 | -1.000 | 1.000 | 17.04 | 29.81 | 3.15 | 3025 |
| 7 | -1.000 | 1.000 | 1.000 | -1.000 | 22.22 | 28.47 | 3.28 | 3936 |
| 8 | -1.000 | 1.000 | 1.000 | 1.000 | 22.22 | 28.47 | 3.28 | 3936 |
| 9 | 1.000 | -1.000 | -1.000 | -1.000 | 17.01 | 29.81 | 3.15 | 3019 |
| 10 | 1.000 | -1.000 | -1.000 | 1.000 | 17.01 | 29.81 | 3.15 | 3019 |
| 11 | 1.000 | -1.000 | 1.000 | -1.000 | 22.22 | 28.47 | 3.28 | 3936 |
| 12 | 1.000 | -1.000 | 1.000 | 1.000 | 22.22 | 28.47 | 3.28 | 3936 |
| 13 | 1.000 | 1.000 | -1.000 | -1.000 | 17.04 | 29.81 | 3.15 | 3025 |
| 14 | 1.000 | 1.000 | -1.000 | 1.000 | 17.04 | 29.81 | 3.15 | 3025 |
| 15 | 1.000 | 1.000 | 1.000 | -1.000 | 22.22 | 28.47 | 3.28 | 3936 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 | 22.22 | 28.47 | 3.28 | 3936 |
| 17 | -1.414 | 0.000 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |
| 18 | 1.414 | 0.000 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |
| 19 | 0.000 | -1.414 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |
| 20 | 0.000 | 1.414 | 0.000 | 0.000 | 20.05 | 28.23 | 3.15 | 3386 |
| 21 | 0.000 | 0.000 | -1.414 | 0.000 | 15.77 | 30.02 | 2.99 | 2692 |
| 22 | 0.000 | 0.000 | 1.414 | 0.000 | 21.95 | 28.55 | 3.28 | 3896 |
| 23 | 0.000 | 0.000 | 0.000 | -1.414 | 20.06 | 28.23 | 3.15 | 3388 |
| 24 | 0.000 | 0.000 | 0.000 | 1.414 | 20.06 | 28.23 | 3.15 | 3388 |
| 25 | 0.000 | 0.000 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |

Table E2 - Estimated second-order response surface for experiment 3, variable Y1.

| | |
|---|---|
| RESPONSE MEAN | 19.6836 |
| ROOT MSE | 0.23694697 |
| R-SQUARE | 0.99566292 |
| COEF OF VARIATION | 0.01203779 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 126.5053 | 0.9772 | 563.31 | 0.0001 |
| QUADRATIC | 4 | 2.3832 | 0.0184 | 10.61 | 0.0013 |
| CROSSPRODUCT | 6 | 0.00090000 | 0.0000 | 0.00 | 1.0000 |
| TOTAL REGRESS | 14 | 128.8893 | 0.9957 | 163.98 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 0.56143865 | 0.05614386 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 19.9543 | 0.14213765 | 140.39 | 0.0001 |
| X1 | 1 | 0 | 0.05298455 | 0.00 | 1.0000 |
| X2 | 1 | 0.00529332 | 0.05298455 | 0.10 | 0.9224 |
| X3 | 1 | 2.5151 | 0.05298455 | 47.47 | 0.0001 |
| X4 | 1 | 0 | 0.05298455 | 0.00 | 1.0000 |
| X1*X1 | 1 | 0.06606482 | 0.08378859 | 0.79 | 0.4487 |
| X1*X2 | 1 | 0 | 0.05923674 | 0.00 | 1.0000 |
| X2*X2 | 1 | 0.06356407 | 0.08378859 | 0.76 | 0.4656 |
| X1*X3 | 1 | 0 | 0.05923674 | 0.00 | 1.0000 |
| X2*X3 | 1 | -0.00750000 | 0.05923674 | -0.13 | 0.9018 |
| X3*X3 | 1 | -0.53411643 | 0.08378859 | -6.37 | 0.0001 |
| X1*X4 | 1 | 0 | 0.05923674 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.05923674 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.05923674 | 0.00 | 1.0000 |
| X4*X4 | 1 | 0.06606482 | 0.08378859 | 0.79 | 0.4487 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.03490383 | 0.00698077 | 0.12 | 0.9836 |
| X2 | 5 | 0.03377176 | 0.00675435 | 0.12 | 0.9847 |
| X3 | 5 | 128.7870 | 25.7574 | 458.78 | 0.0001 |
| X4 | 5 | 0.03490383 | 0.00698077 | 0.12 | 0.9836 |

Table E3 - Estimated second-order response surface for experiment 3, variable Y2.

| | |
|---|---|
| RESPONSE MEAN | 28.7792 |
| ROOT MSE | 0.26649547 |
| R-SQUARE | 0.94853256 |
| COEF OF VARIATION | 0.00926000 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 8.9704 | 0.6501 | 31.58 | 0.0001 |
| QUADRATIC | 4 | 3.0993 | 0.2246 | 10.91 | 0.0011 |
| CROSSPRODUCT | 6 | 1.0191 | 0.0738 | 2.39 | 0.1070 |
| TOTAL REGRESS | 14 | 13.0888 | 0.9485 | 13.16 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 0.71019838 | 0.07101984 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 28.1029 | 0.15986295 | 175.79 | 0.0001 |
| X1 | 1 | 0.14700888 | 0.05959200 | 2.47 | 0.0333 |
| X2 | 1 | 0.14700888 | 0.05959200 | 2.47 | 0.0333 |
| X3 | 1 | -0.62996705 | 0.05959200 | -10.57 | 0.0001 |
| X4 | 1 | -0.09200556 | 0.05959200 | -1.54 | 0.1536 |
| X1*X1 | 1 | 0.07942666 | 0.09423746 | 0.84 | 0.4190 |
| X1*X2 | 1 | -0.18375000 | 0.06662387 | -2.76 | 0.0202 |
| X2*X2 | 1 | 0.07942666 | 0.09423746 | 0.84 | 0.4190 |
| X1*X3 | 1 | -0.01250000 | 0.06662387 | -0.19 | 0.8549 |
| X2*X3 | 1 | -0.01250000 | 0.06662387 | -0.19 | 0.8549 |
| X3*X3 | 1 | 0.60708601 | 0.09423746 | 6.44 | 0.0001 |
| X1*X4 | 1 | 0.11500000 | 0.06662387 | 1.73 | 0.1150 |
| X2*X4 | 1 | 0.11500000 | 0.06662387 | 1.73 | 0.1150 |
| X3*X4 | 1 | -0.05625000 | 0.06662387 | -0.84 | 0.4182 |
| X4*X4 | 1 | 0.07942666 | 0.09423746 | 0.84 | 0.4190 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 1.2370 | 0.24739631 | 3.48 | 0.0441 |
| X2 | 5 | 1.2370 | 0.24739631 | 3.48 | 0.0441 |
| X3 | 5 | 10.9397 | 2.1879 | 30.81 | 0.0001 |
| X4 | 5 | 0.69356569 | 0.13871314 | 1.95 | 0.1720 |

Table E4 - Estimated second-order response surface for experiment 3, variable Y3.

| | |
|---|---|
| RESPONSE MEAN | 3.1904 |
| ROOT MSE | 0.02805514 |
| R-SQUARE | 0.94250446 |
| COEF OF VARIATION | 0.00879361 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 0.10514005 | 0.7680 | 33.40 | 0.0001 |
| QUADRATIC | 4 | 0.02388504 | 0.1745 | 7.59 | 0.0045 |
| CROSSPRODUCT | 6 | 0 | 0.0000 | 0.00 | 1.0000 |
| TOTAL REGRESS | 14 | 0.12902509 | 0.9425 | 11.71 | 0.0002 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 0.00787091 | 0.00078709 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3.1036 | 0.01682947 | 184.42 | 0.0001 |
| X1 | 1 | 6.43968E-16 | 0.00627351 | 0.00 | 1.0000 |
| X2 | 1 | 0 | 0.00627351 | 0.00 | 1.0000 |
| X3 | 1 | 0.07250738 | 0.00627351 | 11.56 | 0.0001 |
| X4 | 1 | 0 | 0.00627351 | 0.00 | 1.0000 |
| X1*X1 | 1 | 0.02899740 | 0.00992079 | 2.92 | 0.0152 |
| X1*X2 | 1 | 0 | 0.00701379 | 0.00 | 1.0000 |
| X2*X2 | 1 | 0.02899740 | 0.00992079 | 2.92 | 0.0152 |
| X1*X3 | 1 | 0 | 0.00701379 | 0.00 | 1.0000 |
| X2*X3 | 1 | 0 | 0.00701379 | 0.00 | 1.0000 |
| X3*X3 | 1 | 0.02149514 | 0.00992079 | 2.17 | 0.0555 |
| X1*X4 | 1 | 0 | 0.00701379 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.00701379 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.00701379 | 0.00 | 1.0000 |
| X4*X4 | 1 | 0.02899740 | 0.00992079 | 2.92 | 0.0152 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.00672436 | 0.00134487 | 1.71 | 0.2201 |
| X2 | 5 | 0.00672436 | 0.00134487 | 1.71 | 0.2201 |
| X3 | 5 | 0.10883504 | 0.02176701 | 27.66 | 0.0001 |
| X4 | 5 | 0.00672436 | 0.00134487 | 1.71 | 0.2201 |

Table E5 - Estimated second-order response surface for experiment 3, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 3425.2800 |
| ROOT MSE | 47.8332 |
| R-SQUARE | 0.99445230 |
| COEF OF VARIATION | 0.01396475 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 4033035 | 0.9779 | 440.67 | 0.0001 |
| QUADRATIC | 4 | 52760.8754 | 0.0128 | 5.76 | 0.0114 |
| CROSSPRODUCT | 6 | 15583.5000 | 0.0038 | 1.14 | 0.4087 |
| TOTAL REGRESS | 14 | 4101379 | 0.9945 | 128.04 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 22880.1430 | 2288.0143 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3312.7381 | 28.6938 | 115.45 | 0.0001 |
| X1 | 1 | 16.7010 | 10.6962 | 1.56 | 0.1495 |
| X2 | 1 | 17.7597 | 10.6962 | 1.66 | 0.1278 |
| X3 | 1 | 448.2499 | 10.6962 | 41.91 | 0.0001 |
| X4 | 1 | -11.9007 | 10.6962 | -1.11 | 0.2919 |
| X1*X1 | 1 | 47.0501 | 16.9147 | 2.78 | 0.0194 |
| X1*X2 | 1 | -20.8750 | 11.9583 | -1.75 | 0.1115 |
| X2*X2 | 1 | 46.5499 | 16.9147 | 2.75 | 0.0204 |
| X1*X3 | 1 | 3.1250 | 11.9583 | 0.26 | 0.7991 |
| X2*X3 | 1 | 1.6250 | 11.9583 | 0.14 | 0.8946 |
| X3*X3 | 1 | 0.03585625 | 16.9147 | 0.00 | 0.9984 |
| X1*X4 | 1 | 14.8750 | 11.9583 | 1.24 | 0.2419 |
| X2*X4 | 1 | 14.8750 | 11.9583 | 1.24 | 0.2419 |
| X3*X4 | 1 | -9.1250 | 11.9583 | -0.76 | 0.4630 |
| X4*X4 | 1 | 47.0501 | 16.9147 | 2.78 | 0.0194 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 33950.1284 | 6790.0257 | 2.97 | 0.0674 |
| X2 | 5 | 34191.3528 | 6838.2706 | 2.99 | 0.0662 |
| X3 | 5 | 4019847 | 803969 | 351.38 | 0.0001 |
| X4 | 5 | 28948.3625 | 5789.6725 | 2.53 | 0.0992 |

# APPENDIX F

## Data and detailed analysis for Experiment 4

Table F1 - Treatment combinations and response data for experiment 4.

| OBS | X1 | X2 | X3 | X4 | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|---|---|---|---|
| 1 | -1.000 | -1.000 | -1.000 | -1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 2 | -1.000 | -1.000 | -1.000 | 1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 3 | -1.000 | -1.000 | 1.000 | -1.000 | 18.10 | 27.40 | 3.15 | 2963 |
| 4 | -1.000 | -1.000 | 1.000 | 1.000 | 18.10 | 27.40 | 3.15 | 2963 |
| 5 | -1.000 | 1.000 | -1.000 | -1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 6 | -1.000 | 1.000 | -1.000 | 1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 7 | -1.000 | 1.000 | 1.000 | -1.000 | 18.03 | 27.50 | 2.99 | 2813 |
| 8 | -1.000 | 1.000 | 1.000 | 1.000 | 18.03 | 27.50 | 2.99 | 2813 |
| 9 | 1.000 | -1.000 | -1.000 | -1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 10 | 1.000 | -1.000 | -1.000 | 1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 11 | 1.000 | -1.000 | 1.000 | -1.000 | 18.10 | 27.40 | 3.15 | 2963 |
| 12 | 1.000 | -1.000 | 1.000 | 1.000 | 18.10 | 27.40 | 3.15 | 2963 |
| 13 | 1.000 | 1.000 | -1.000 | -1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 14 | 1.000 | 1.000 | -1.000 | 1.000 | 18.45 | 27.82 | 2.54 | 2468 |
| 15 | 1.000 | 1.000 | 1.000 | -1.000 | 18.03 | 27.50 | 2.99 | 2813 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 | 18.03 | 27.50 | 2.99 | 2813 |
| 17 | -1.414 | 0.000 | 0.000 | 0.000 | 17.00 | 27.65 | 2.79 | 2481 |
| 18 | 1.414 | 0.000 | 0.000 | 0.000 | 17.00 | 27.65 | 2.79 | 2481 |
| 19 | 0.000 | -1.414 | 0.000 | 0.000 | 17.00 | 27.65 | 2.79 | 2481 |
| 20 | 0.000 | 1.414 | 0.000 | 0.000 | 17.00 | 27.65 | 2.79 | 2481 |
| 21 | 0.000 | 0.000 | -1.414 | 0.000 | 19.68 | 27.81 | 2.54 | 2626 |
| 22 | 0.000 | 0.000 | 1.414 | 0.000 | 16.25 | 27.55 | 3.15 | 2672 |
| 23 | 0.000 | 0.000 | 0.000 | -1.414 | 17.00 | 27.65 | 2.79 | 2481 |
| 24 | 0.000 | 0.000 | 0.000 | 1.414 | 17.00 | 27.65 | 2.79 | 2481 |
| 25 | 0.000 | 0.000 | 0.000 | 0.000 | 17.00 | 27.65 | 2.79 | 2481 |

Table F2 - Estimated second-order response surface for experiment 4, variable Y1.

| | |
|---|---|
| RESPONSE MEAN | 17.8820 |
| ROOT MSE | 0.60957060 |
| R-SQUARE | 0.73833391 |
| COEF OF VARIATION | 0.03408850 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 3.1484 | 0.2217 | 2.12 | 0.1531 |
| QUADRATIC | 4 | 7.3314 | 0.5163 | 4.93 | 0.0186 |
| CROSSPRODUCT | 6 | 0.00490000 | 0.0003 | 0.00 | 1.0000 |
| TOTAL REGRESS | 14 | 10.4846 | 0.7383 | 2.02 | 0.1342 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 3.7158 | 0.37157632 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 16.5043 | 0.36566383 | 45.14 | 0.0001 |
| X1 | 1 | 0 | 0.13630825 | 0.00 | 1.0000 |
| X2 | 1 | -0.01400085 | 0.13630825 | -0.10 | 0.9202 |
| X3 | 1 | -0.39652495 | 0.13630825 | -2.91 | 0.0156 |
| X4 | 1 | 0 | 0.13630825 | 0.00 | 1.0000 |
| X1*X1 | 1 | 0.30990425 | 0.21555483 | 1.44 | 0.1811 |
| X1*X2 | 1 | 0 | 0.15239265 | 0.00 | 1.0000 |
| X2*X2 | 1 | 0.30990425 | 0.21555483 | 1.44 | 0.1811 |
| X1*X3 | 1 | 0 | 0.15239265 | 0.00 | 1.0000 |
| X2*X3 | 1 | -0.01750000 | 0.15239265 | -0.11 | 0.9108 |
| X3*X3 | 1 | 0.79255001 | 0.21555483 | 3.68 | 0.0043 |
| X1*X4 | 1 | 0 | 0.15239265 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.15239265 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.15239265 | 0.00 | 1.0000 |
| X4*X4 | 1 | 0.30990425 | 0.21555483 | 1.44 | 0.1811 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.76804664 | 0.15360933 | 0.41 | 0.8292 |
| X2 | 5 | 0.77686688 | 0.15537338 | 0.42 | 0.8260 |
| X3 | 5 | 8.1726 | 1.6345 | 4.40 | 0.0223 |
| X4 | 5 | 0.76804664 | 0.15360933 | 0.41 | 0.8292 |

Table F3 - Estimated second-order response surface for experiment 4, variable Y2.

| | |
|---|---|
| RESPONSE MEAN | 27.6428 |
| ROOT MSE | 0.05502935 |
| R-SQUARE | 0.94998829 |
| COEF OF VARIATION | 0.00199073 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 0.56169332 | 0.9276 | 46.37 | 0.0001 |
| QUADRATIC | 4 | 0.00352838 | 0.0058 | 0.29 | 0.8771 |
| CROSSPRODUCT | 6 | 0.01000000 | 0.0165 | 0.55 | 0.7600 |
| TOTAL REGRESS | 14 | 0.57522171 | 0.9500 | 13.57 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 0.03028229 | 0.00302823 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 27.6692 | 0.03301052 | 838.19 | 0.0001 |
| X1 | 1 | 0 | 0.01230531 | 0.00 | 1.0000 |
| X2 | 1 | 0.02000121 | 0.01230531 | 1.63 | 0.1351 |
| X3 | 1 | -0.16639205 | 0.01230531 | -13.52 | 0.0001 |
| X4 | 1 | 0 | 0.01230531 | 0.00 | 1.0000 |
| X1*X1 | 1 | -0.01200036 | 0.01945934 | -0.62 | 0.5512 |
| X1*X2 | 1 | 0 | 0.01375734 | 0.00 | 1.0000 |
| X2*X2 | 1 | -0.01200036 | 0.01945934 | -0.62 | 0.5512 |
| X1*X3 | 1 | 0 | 0.01375734 | 0.00 | 1.0000 |
| X2*X3 | 1 | 0.02500000 | 0.01375734 | 1.82 | 0.0992 |
| X3*X3 | 1 | 0.00300417 | 0.01945934 | 0.15 | 0.8804 |
| X1*X4 | 1 | 0 | 0.01375734 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.01375734 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.01375734 | 0.00 | 1.0000 |
| X4*X4 | 1 | -0.01200036 | 0.01945934 | -0.62 | 0.5512 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.00115165 | 0.00023033 | 0.08 | 0.9946 |
| X2 | 5 | 0.01915214 | 0.00383043 | 1.26 | 0.3505 |
| X3 | 5 | 0.56376502 | 0.11275300 | 37.23 | 0.0001 |
| X4 | 5 | 0.00115165 | 0.00023033 | 0.08 | 0.9946 |

Table F4 - Estimated second-order response surface for experiment 4, variable Y3.

| | |
|---|---|
| RESPONSE MEAN | 2.8040 |
| ROOT MSE | 0.03604781 |
| R-SQUARE | 0.99048444 |
| COEF OF VARIATION | 0.01285585 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 1.3224 | 0.9683 | 254.41 | 0.0001 |
| QUADRATIC | 4 | 0.00464996 | 0.0034 | 0.89 | 0.5021 |
| CROSSPRODUCT | 6 | 0.02560000 | 0.0187 | 3.28 | 0.0472 |
| TOTAL REGRESS | 14 | 1.3526 | 0.9905 | 74.35 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 0.01299444 | 0.00129944 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 2.7980 | 0.02162404 | 129.39 | 0.0001 |
| X1 | 1 | 5.32939E-16 | 0.00806078 | 0.00 | 1.0000 |
| X2 | 1 | -0.03200193 | 0.00806078 | -3.97 | 0.0026 |
| X3 | 1 | 0.25514241 | 0.00806078 | 31.65 | 0.0001 |
| X4 | 1 | 0 | 0.00806078 | 0.00 | 1.0000 |
| X1*X1 | 1 | -0.00500272 | 0.01274713 | -0.39 | 0.7030 |
| X1*X2 | 1 | 0 | 0.00901195 | 0.00 | 1.0000 |
| X2*X2 | 1 | -0.00500272 | 0.01274713 | -0.39 | 0.7030 |
| X1*X3 | 1 | 0 | 0.00901195 | 0.00 | 1.0000 |
| X2*X3 | 1 | -0.04000000 | 0.00901195 | -4.44 | 0.0013 |
| X3*X3 | 1 | 0.02250559 | 0.01274713 | 1.77 | 0.1079 |
| X1*X4 | 1 | 0 | 0.00901195 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.00901195 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.00901195 | 0.00 | 1.0000 |
| X4*X4 | 1 | -0.00500272 | 0.01274713 | -0.39 | 0.7030 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 0.00020014 | .00004002899 | 0.03 | 0.9994 |
| X2 | 5 | 0.04628138 | 0.00925628 | 7.12 | 0.0044 |
| X3 | 5 | 1.3315 | 0.26630498 | 204.94 | 0.0001 |
| X4 | 5 | 0.00020014 | .00004002899 | 0.03 | 0.9994 |

Table F5 - Estimated second-order response surface for experiment 4, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 2620.5200 |
| ROOT MSE | 115.2110 |
| R-SQUARE | 0.85915199 |
| COEF OF VARIATION | 0.04396494 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LINEAR | 4 | 604583 | 0.6415 | 11.39 | 0.0010 |
| QUADRATIC | 4 | 182586 | 0.1937 | 3.44 | 0.0515 |
| CROSSPRODUCT | 6 | 22500.0000 | 0.0239 | 0.28 | 0.9323 |
| TOTAL REGRESS | 14 | 809668 | 0.8592 | 4.36 | 0.0122 |

| RESIDUAL | DF | SS | MEAN SQUARE |
|---|---|---|---|
| TOTAL ERROR | 10 | 132736 | 13273.5762 |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 2408.7224 | 69.1118 | 34.85 | 0.0001 |
| X1 | 1 | 0 | 25.7627 | 0.00 | 1.0000 |
| X2 | 1 | -30.0018 | 25.7627 | -1.16 | 0.2712 |
| X3 | 1 | 171.2625 | 25.7627 | 6.65 | 0.0001 |
| X4 | 1 | 0 | 25.7627 | 0.00 | 1.0000 |
| X1*X1 | 1 | 45.1844 | 40.7406 | 1.11 | 0.2934 |
| X1*X2 | 1 | 0 | 28.8028 | 0.00 | 1.0000 |
| X2*X2 | 1 | 45.1844 | 40.7406 | 1.11 | 0.2934 |
| X1*X3 | 1 | 0 | 28.8028 | 0.00 | 1.0000 |
| X2*X3 | 1 | -37.5000 | 28.8028 | -1.30 | 0.2221 |
| X3*X3 | 1 | 129.2098 | 40.7406 | 3.17 | 0.0100 |
| X1*X4 | 1 | 0 | 28.8028 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 28.8028 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 28.8028 | 0.00 | 1.0000 |
| X4*X4 | 1 | 45.1844 | 40.7406 | 1.11 | 0.2934 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 16327.1122 | 3265.4224 | 0.25 | 0.9325 |
| X2 | 5 | 56828.1995 | 11365.6399 | 0.86 | 0.5415 |
| X3 | 5 | 742595 | 148519 | 11.19 | 0.0008 |
| X4 | 5 | 16327.1122 | 3265.4224 | 0.25 | 0.9325 |

# APPENDIX G

## Data and detailed analysis for Experiment 5

Table G1 - Treatment combinations and response data for experiment 5.

| OBS | F | FSQ | X1 | X2 | X3 | X4 | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | -1.000 | -1.000 | -1.000 | -1.000 | 22.70 | 33.59 | 1.36 | 1967 |
| 2 | 1 | 1 | 1.000 | 1.000 | -1.000 | -1.000 | 21.34 | 34.88 | 3.76 | 5306 |
| 3 | 1 | 1 | 1.000 | -1.000 | 1.000 | -1.000 | 21.58 | 34.52 | 2.23 | 3148 |
| 4 | 1 | 1 | 1.000 | -1.000 | -1.000 | 1.000 | 23.17 | 32.80 | 1.36 | 1958 |
| 5 | 1 | 1 | -1.000 | 1.000 | 1.000 | -1.000 | 21.33 | 33.39 | 3.73 | 5040 |
| 6 | 1 | 1 | -1.000 | 1.000 | -1.000 | 1.000 | 21.89 | 33.87 | 2.79 | 3926 |
| 7 | 1 | 1 | -1.000 | -1.000 | 1.000 | 1.000 | 23.53 | 34.88 | 0.75 | 1171 |
| 8 | 1 | 1 | 1.000 | 1.000 | 1.000 | 1.000 | 21.21 | 34.67 | 3.63 | 5053 |
| 9 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 10 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 11 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 12 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 13 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 14 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 15 | 2 | 0 | 1.000 | -1.000 | -1.000 | -1.000 | 20.74 | 32.70 | 2.23 | 2864 |
| 16 | 2 | 0 | -1.000 | 1.000 | -1.000 | -1.000 | 20.51 | 32.70 | 3.58 | 4557 |
| 17 | 2 | 0 | -1.000 | -1.000 | 1.000 | -1.000 | 21.16 | 32.70 | 1.84 | 2413 |
| 18 | 2 | 0 | -1.000 | -1.000 | -1.000 | 1.000 | 23.17 | 32.69 | 0.75 | 1080 |
| 19 | 2 | 0 | 1.000 | 1.000 | 1.000 | -1.000 | 21.19 | 32.70 | 3.79 | 4978 |
| 20 | 2 | 0 | 1.000 | 1.000 | -1.000 | 1.000 | 20.00 | 32.70 | 3.63 | 4496 |
| 21 | 2 | 0 | 1.000 | -1.000 | 1.000 | 1.000 | 21.00 | 32.70 | 1.84 | 2394 |
| 22 | 2 | 0 | -1.000 | 1.000 | 1.000 | 1.000 | 20.08 | 32.63 | 3.15 | 3913 |
| 23 | 3 | 1 | -1.712 | 0.000 | 0.000 | 0.000 | 19.98 | 28.28 | 2.23 | 2398 |
| 24 | 3 | 1 | 1.712 | 0.000 | 0.000 | 0.000 | 20.50 | 28.19 | 3.28 | 3932 |
| 25 | 3 | 1 | 0.000 | -1.712 | 0.000 | 0.000 | 20.35 | 28.46 | 1.36 | 1502 |
| 26 | 3 | 1 | 0.000 | 1.712 | 0.000 | 0.000 | 20.53 | 28.17 | 3.76 | 4131 |
| 27 | 3 | 1 | 0.000 | 0.000 | -1.712 | 0.000 | 20.22 | 27.99 | 2.54 | 2737 |
| 28 | 3 | 1 | 0.000 | 0.000 | 1.712 | 0.000 | 20.54 | 28.23 | 3.15 | 3468 |
| 29 | 3 | 1 | 0.000 | 0.000 | 0.000 | -1.712 | 20.87 | 28.49 | 3.63 | 4093 |
| 30 | 3 | 1 | 0.000 | 0.000 | 0.000 | 1.712 | 19.92 | 27.99 | 2.54 | 2698 |

Table G2 - Estimated second-order response surface for experiment 5, variable Y1.

| | |
|---|---|
| RESPONSE MEAN | 21.0710 |
| ROOT MSE | 0.56866813 |
| R-SQUARE | 0.85219756 |
| COEF OF VARIATION | 0.02698819 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 12.4233 | 0.4368 | 19.21 | 0.0010 |
| LINEAR | 4 | 4.6721 | 0.1643 | 3.61 | 0.0343 |
| QUADRATIC | 4 | 2.6152 | 0.0919 | 2.02 | 0.1506 |
| CROSSPRODUCT | 6 | 4.5287 | 0.1592 | 2.33 | 0.0944 |
| TOTAL REGRESS | 16 | 24.2393 | 0.8522 | 4.68 | 0.0038 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 4.2040 | 0.46710942310050.840 | | 0.0001 |
| PURE ERROR 1) | 4 | 1.70530E-13 | 4.26326E-14 | | |
| TOTAL ERROR | 13 | 4.2040 | 0.32338344 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 21.7386 | 0.37125626 | 58.55 | 0.0001 |
| X1 | 1 | -0.14864956 | 0.12162282 | -1.22 | 0.2433 |
| X2 | 1 | -0.42045042 | 0.12162282 | -3.46 | 0.0043 |
| X3 | 1 | -0.08655062 | 0.12162282 | -0.71 | 0.4893 |
| X4 | 1 | 0.08570166 | 0.12162282 | 0.70 | 0.4935 |
| X1*X1 | 1 | 0.14965311 | 0.13692524 | 1.09 | 0.2943 |
| X1*X2 | 1 | 0.25000000 | 0.14216703 | 1.76 | 0.1022 |
| X2*X2 | 1 | 0.21789051 | 0.13692524 | 1.59 | 0.1356 |
| X1*X3 | 1 | 0.11875000 | 0.14216703 | 0.84 | 0.4186 |
| X2*X3 | 1 | 0.16125000 | 0.14216703 | 1.13 | 0.2772 |
| X3*X3 | 1 | 0.19741929 | 0.13692524 | 1.44 | 0.1730 |
| X1*X4 | 1 | -0.15250000 | 0.14216703 | -1.07 | 0.3029 |
| X2*X4 | 1 | -0.36750000 | 0.14216703 | -2.58 | 0.0226 |
| X3*X4 | 1 | -0.14875000 | 0.14216703 | -1.05 | 0.3145 |
| X4*X4 | 1 | 0.20253709 | 0.13692524 | 1.48 | 0.1629 |
| F | 1 | -0.76567608 | 0.13212081 | -5.80 | 0.0001 |
| FSQ | 1 | 0.35977847 | 0.21645878 | 1.66 | 0.1204 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 2.4671 | 0.49341964 | 1.53 | 0.2487 |
| X2 | 5 | 8.2605 | 1.6521 | 5.11 | 0.0083 |
| X3 | 5 | 1.8317 | 0.36633825 | 1.13 | 0.3913 |
| X4 | 5 | 3.7552 | 0.75103003 | 2.32 | 0.1026 |

1) See footnote at bottom of Table B2.

Table G3 - Estimated second-order response surface for experiment 5, variable Y2.

| | |
|---|---|
| RESPONSE MEAN | 32.0137 |
| ROOT MSE | 0.23697804 |
| R-SQUARE | 0.99575055 |
| COEF OF VARIATION | 0.00740240 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 167.3271 | 0.9740 | 1489.77 | 0.0001 |
| LINEAR | 4 | 0.44304738 | 0.0026 | 1.97 | 0.1583 |
| QUADRATIC | 4 | 0.04790909 | 0.0003 | 0.21 | 0.9264 |
| CROSSPRODUCT | 6 | 3.2534 | 0.0189 | 9.66 | 0.0004 |
| TOTAL REGRESS | 16 | 171.0714 | 0.9958 | 190.39 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 0.73006171 | 0.0811179738 | 1519.493 | 0.0001 |
| PURE ERROR 1) | 4 | 2.27374E-13 | 5.68434E-14 | | |
| TOTAL ERROR | 13 | 0.73006171 | 0.05615859 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 38.5812 | 0.15471165 | 249.37 | 0.0001 |
| X1 | 1 | 0.04875700 | 0.05068323 | 0.96 | 0.3536 |
| X2 | 1 | 0.02120219 | 0.05068323 | 0.42 | 0.6825 |
| X3 | 1 | 0.12217060 | 0.05068323 | 2.41 | 0.0315 |
| X4 | 1 | -0.05013291 | 0.05068323 | -0.99 | 0.3406 |
| X1*X1 | 1 | -0.00471313 | 0.05706013 | -0.08 | 0.9354 |
| X1*X2 | 1 | 0.21875000 | 0.05924451 | 3.69 | 0.0027 |
| X2*X2 | 1 | 0.02258183 | 0.05706013 | 0.40 | 0.6987 |
| X1*X3 | 1 | 0.04750000 | 0.05924451 | 0.80 | 0.4371 |
| X2*X3 | 1 | -0.23625000 | 0.05924451 | -3.99 | 0.0015 |
| X3*X3 | 1 | -0.04736151 | 0.05706013 | -0.83 | 0.4215 |
| X1*X4 | 1 | -0.22625000 | 0.05924451 | -3.82 | 0.0021 |
| X2*X4 | 1 | 0.04000000 | 0.05924451 | 0.68 | 0.5114 |
| X3*X4 | 1 | 0.21125000 | 0.05924451 | 3.57 | 0.0035 |
| X4*X4 | 1 | -0.00300719 | 0.05706013 | -0.05 | 0.9588 |
| F | 1 | -2.9324 | 0.05505800 | -53.26 | 0.0001 |
| FSQ | 1 | -1.5351 | 0.09020372 | -17.02 | 0.0001 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 1.6731 | 0.33462084 | 5.96 | 0.0044 |
| X2 | 5 | 1.7029 | 0.34057466 | 6.06 | 0.0041 |
| X3 | 5 | 2.0081 | 0.40162866 | 7.15 | 0.0020 |
| X4 | 5 | 1.6138 | 0.32275033 | 5.75 | 0.0052 |

1) See footnote at bottom of Table B2.

Table G4 - Estimated second-order response surface for experiment 5, variable Y3.

| | |
|---|---|
| RESPONSE MEAN | 2.6660 |
| ROOT MSE | 0.22857232 |
| R-SQUARE | 0.97222054 |
| COEF OF VARIATION | 0.08573605 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 0.64746886 | 0.0265 | 6.20 | 0.5394 |
| LINEAR | 4 | 21.9946 | 0.8996 | 105.25 | 0.0001 |
| QUADRATIC | 4 | 0.72808977 | 0.0298 | 3.48 | 0.0383 |
| CROSSPRODUCT | 6 | 0.39997500 | 0.0164 | 1.28 | 0.3332 |
| TOTAL REGRESS | 16 | 23.7701 | 0.9722 | 28.44 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 0.67918895 | 0.07546544 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 4 | 0 | 0 | | |
| TOTAL ERROR | 13 | 0.67918895 | 0.05224530 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 2.6452 | 0.14922395 | 17.73 | 0.0001 |
| X1 | 1 | 0.28897779 | 0.04888547 | 5.91 | 0.0001 |
| X2 | 1 | 0.90608826 | 0.04888547 | 18.53 | 0.0001 |
| X3 | 1 | 0.11638153 | 0.04888547 | 2.38 | 0.0333 |
| X4 | 1 | -0.29668435 | 0.04888547 | -6.07 | 0.0001 |
| X1*X1 | 1 | -0.09887927 | 0.05503618 | -1.80 | 0.0957 |
| X1*X2 | 1 | -0.08750000 | 0.05714308 | -1.53 | 0.1497 |
| X2*X2 | 1 | -0.16541073 | 0.05503618 | -3.01 | 0.0101 |
| X1*X3 | 1 | -0.03000000 | 0.05714308 | -0.52 | 0.6084 |
| X2*X3 | 1 | -0.02625000 | 0.05714308 | -0.46 | 0.6536 |
| X3*X3 | 1 | -0.06817244 | 0.05503618 | -1.24 | 0.2374 |
| X1*X4 | 1 | 0.09500000 | 0.05714308 | 1.66 | 0.1203 |
| X2*X4 | 1 | 0.08125000 | 0.05714308 | 1.42 | 0.1786 |
| X3*X4 | 1 | 0.01125000 | 0.05714308 | 0.20 | 0.8470 |
| X4*X4 | 1 | 0.01371244 | 0.05503618 | 0.25 | 0.8071 |
| F | 1 | 0.16876844 | 0.05310507 | 3.18 | 0.0073 |
| FSQ | 1 | -0.10668611 | 0.08700415 | -1.23 | 0.2419 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 2.2756 | 0.45511726 | 8.71 | 0.0008 |
| X2 | 5 | 18.6596 | 3.7319 | 71.43 | 0.0001 |
| X3 | 5 | 0.40372384 | 0.08074477 | 1.55 | 0.2432 |
| X4 | 5 | 2.1796 | 0.43592233 | 8.34 | 0.0010 |

1) See footnote at bottom of Table B2.

Table G5 - Estimated second-order response surface for experiment 5, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 3385.8667 |
| ROOT MSE | 363.2595 |
| R-SQUARE | 0.95622982 |
| COEF OF VARIATION | 0.10728701 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 779588 | 0.0199 | 2.95 | 0.0001 |
| LINEAR | 4 | 35492141 | 0.9056 | 67.24 | 0.0001 |
| QUADRATIC | 4 | 827477 | 0.0211 | 1.57 | 0.2411 |
| CROSSPRODUCT | 6 | 377494 | 0.0096 | 0.48 | 0.8140 |
| TOTAL REGRESS | 16 | 37476700 | 0.9562 | 17.75 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 1715447 | 190605 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 4 | 0 | 0 | | |
| TOTAL ERROR | 13 | 1715447 | 131957 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 4086.3912 | 237.1548 | 17.23 | 0.0001 |
| X1 | 1 | 400.5239 | 77.6914 | 5.16 | 0.0002 |
| X2 | 1 | 1133.2438 | 77.6914 | 14.59 | 0.0001 |
| X3 | 1 | 146.7152 | 77.6914 | 1.89 | 0.0815 |
| X4 | 1 | -396.5915 | 77.6914 | -5.10 | 0.0002 |
| X1*X1 | 1 | -68.1039 | 87.4665 | -0.78 | 0.4501 |
| X1*X2 | 1 | -83.5000 | 90.8149 | -0.92 | 0.3746 |
| X2*X2 | 1 | -187.0076 | 87.4665 | -2.14 | 0.0521 |
| X1*X3 | 1 | -3.6250 | 90.8149 | -0.04 | 0.9688 |
| X2*X3 | 1 | -34.8750 | 90.8149 | -0.38 | 0.7072 |
| X3*X3 | 1 | -89.4281 | 87.4665 | -1.02 | 0.3252 |
| X1*X4 | 1 | 93.2500 | 90.8149 | 1.03 | 0.3232 |
| X2*X4 | 1 | 81.0000 | 90.8149 | 0.89 | 0.3886 |
| X3*X4 | 1 | 11.6250 | 90.8149 | 0.13 | 0.9001 |
| X4*X4 | 1 | 10.5397 | 87.4665 | 0.12 | 0.9059 |
| F | 1 | -171.0592 | 84.3975 | -2.03 | 0.0637 |
| FSQ | 1 | -208.6047 | 138.2717 | -1.51 | 0.1553 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 3837967 | 767593 | 5.82 | 0.0049 |
| X2 | 5 | 28915145 | 5783029 | 43.82 | 0.0001 |
| X3 | 5 | 630361 | 126072 | 0.96 | 0.4790 |
| X4 | 5 | 3686727 | 737345 | 5.59 | 0.0058 |

1) See footnote at bottom of Table B2.

## APPENDIX H

**Data and detailed analysis for Experiment 6**

Table H1 - Treatment combinations and response data for experiment 6.

| OBS | F | FSQ | X1 | X2 | X3 | X4 | X5 | X6 | Y4 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | -1.000 | -1.000 | -1.000 | -1.000 | -1.000 | -1.000 | 2814 |
| 2 | 1 | 1 | 1.000 | 1.000 | -1.000 | -1.000 | -1.000 | -1.000 | 3572 |
| 3 | 1 | 1 | 1.000 | -1.000 | 1.000 | -1.000 | -1.000 | -1.000 | 3315 |
| 4 | 1 | 1 | -1.000 | 1.000 | 1.000 | -1.000 | -1.000 | -1.000 | 4745 |
| 5 | 1 | 1 | -1.000 | -1.000 | -1.000 | 1.000 | 1.000 | -1.000 | 1789 |
| 6 | 1 | 1 | -1.000 | -1.000 | -1.000 | 1.000 | -1.000 | 1.000 | 3267 |
| 7 | 1 | 1 | -1.000 | -1.000 | -1.000 | -1.000 | 1.000 | 1.000 | 2841 |
| 8 | 1 | 1 | 1.000 | 1.000 | -1.000 | 1.000 | 1.000 | -1.000 | 2272 |
| 9 | 1 | 1 | 1.000 | 1.000 | -1.000 | 1.000 | -1.000 | 1.000 | 4135 |
| 10 | 1 | 1 | 1.000 | 1.000 | -1.000 | -1.000 | 1.000 | 1.000 | 3607 |
| 11 | 1 | 1 | 1.000 | -1.000 | 1.000 | 1.000 | 1.000 | -1.000 | 2110 |
| 12 | 1 | 1 | 1.000 | -1.000 | 1.000 | 1.000 | -1.000 | 1.000 | 3847 |
| 13 | 1 | 1 | 1.000 | -1.000 | 1.000 | -1.000 | 1.000 | 1.000 | 3352 |
| 14 | 1 | 1 | -1.000 | 1.000 | 1.000 | 1.000 | 1.000 | -1.000 | 3031 |
| 15 | 1 | 1 | -1.000 | 1.000 | 1.000 | 1.000 | -1.000 | 1.000 | 5532 |
| 16 | 1 | 1 | -1.000 | 1.000 | 1.000 | -1.000 | 1.000 | 1.000 | 4793 |
| 17 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 18 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 19 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 20 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 21 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 22 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 23 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 24 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 25 | 2 | 0 | 0.000 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 26 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 27 | 2 | 0 | 1.000 | -1.000 | -1.000 | -1.000 | 1.000 | -1.000 | 2444 |
| 28 | 2 | 0 | -1.000 | 1.000 | -1.000 | -1.000 | 1.000 | -1.000 | 3165 |
| 29 | 2 | 0 | -1.000 | -1.000 | 1.000 | -1.000 | 1.000 | -1.000 | 3000 |
| 30 | 2 | 0 | 1.000 | 1.000 | 1.000 | -1.000 | 1.000 | -1.000 | 4171 |
| 31 | 2 | 0 | 1.000 | -1.000 | -1.000 | 1.000 | -1.000 | -1.000 | 2690 |
| 32 | 2 | 0 | 1.000 | -1.000 | -1.000 | 1.000 | 1.000 | 1.000 | 2852 |
| 33 | 2 | 0 | 1.000 | -1.000 | -1.000 | -1.000 | -1.000 | 1.000 | 4454 |
| 34 | 2 | 0 | -1.000 | 1.000 | -1.000 | 1.000 | -1.000 | -1.000 | 3411 |
| 35 | 2 | 0 | -1.000 | 1.000 | -1.000 | 1.000 | 1.000 | 1.000 | 3531 |
| 36 | 2 | 0 | -1.000 | 1.000 | -1.000 | -1.000 | -1.000 | 1.000 | 5761 |
| 37 | 2 | 0 | -1.000 | -1.000 | 1.000 | 1.000 | -1.000 | -1.000 | 3257 |
| 38 | 2 | 0 | -1.000 | -1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 3397 |
| 39 | 2 | 0 | -1.000 | -1.000 | 1.000 | -1.000 | -1.000 | 1.000 | 5512 |
| 40 | 2 | 0 | 1.000 | 1.000 | 1.000 | 1.000 | -1.000 | -1.000 | 4538 |
| 41 | 2 | 0 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 4743 |
| 42 | 2 | 0 | 1.000 | 1.000 | 1.000 | -1.000 | -1.000 | 1.000 | 7608 |
| 43 | 3 | 1 | -2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2713 |
| 44 | 3 | 1 | 2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3648 |
| 45 | 3 | 1 | 0.000 | -2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 2310 |
| 46 | 3 | 1 | 0.000 | 2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 4213 |
| 47 | 3 | 1 | 0.000 | 0.000 | -2.228 | 0.000 | 0.000 | 0.000 | 2370 |
| 48 | 3 | 1 | 0.000 | 0.000 | 2.228 | 0.000 | 0.000 | 0.000 | 3898 |
| 49 | 3 | 1 | 0.000 | 0.000 | 0.000 | -2.228 | 0.000 | 0.000 | 3207 |
| 50 | 3 | 1 | 0.000 | 0.000 | 0.000 | 2.228 | 0.000 | 0.000 | 3257 |
| 51 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | -2.228 | 0.000 | 4438 |
| 52 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 2.228 | 0.000 | 2835 |
| 53 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | -2.228 | 2273 |
| 54 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.228 | 4502 |
| 55 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3388 |

Table H2 - Estimated second-order response surface for experiment 6, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 3633.8727 |
| ROOT MSE | 277.8091 |
| R-SQUARE | 0.96561784 |
| COEF OF VARIATION | 0.07644988 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 5002448 | 0.0891 | 32.41 | 0.0001 |
| LINEAR | 6 | 45802681 | 0.8162 | 98.91 | 0.0001 |
| QUADRATIC | 6 | 337359 | 0.0060 | 0.73 | 0.6310 |
| CROSSPRODUCT | 15 | 3045765 | 0.0543 | 2.63 | 0.0157 |
| TOTAL REGRESS | 29 | 54188252 | 0.9656 | 24.21 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 17 | 1929448 | 113497 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 8 | 0 | 0 | | |
| TOTAL ERROR | 25 | 1929448 | 77177.9053 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 4153.8564 | 136.7171 | 30.38 | 0.0001 |
| X1 | 1 | 46.4411 | 42.9037 | 1.08 | 0.2894 |
| X2 | 1 | 522.6555 | 42.9037 | 12.18 | 0.0001 |
| X3 | 1 | 423.3543 | 42.9037 | 9.87 | 0.0001 |
| X4 | 1 | -253.7829 | 42.9037 | -5.92 | 0.0001 |
| X5 | 1 | -499.2249 | 42.9037 | -11.64 | 0.0001 |
| X6 | 1 | 569.4102 | 42.9037 | 13.27 | 0.0001 |
| X1*X1 | 1 | -24.7878 | 39.5583 | -0.63 | 0.5366 |
| X1*X2 | 1 | 46.5625 | 49.1102 | 0.95 | 0.3521 |
| X2*X2 | 1 | -8.4703 | 39.5583 | -0.21 | 0.8322 |
| X1*X3 | 1 | 30.3125 | 49.1102 | 0.62 | 0.5427 |
| X2*X3 | 1 | 158.3750 | 49.1102 | 3.22 | 0.0035 |
| X3*X3 | 1 | -34.1553 | 39.5583 | -0.86 | 0.3961 |
| X1*X4 | 1 | 2.5000 | 49.1102 | 0.05 | 0.9598 |
| X2*X4 | 1 | -53.3125 | 49.1102 | -1.09 | 0.2880 |
| X3*X4 | 1 | -41.5625 | 49.1102 | -0.85 | 0.4054 |
| X4*X4 | 1 | -14.4131 | 39.5583 | -0.36 | 0.7187 |
| X1*X5 | 1 | 4.5000 | 49.1102 | 0.09 | 0.9277 |
| X2*X5 | 1 | -81.8125 | 49.1102 | -1.67 | 0.1082 |
| X3*X5 | 1 | -67.3125 | 49.1102 | -1.37 | 0.1827 |
| X4*X5 | 1 | 108.0000 | 49.1102 | 2.20 | 0.0373 |
| X5*X5 | 1 | 67.0739 | 39.5583 | 1.70 | 0.1024 |
| X1*X6 | 1 | 2.0000 | 49.1102 | 0.04 | 0.9678 |
| X2*X6 | 1 | 84.4375 | 49.1102 | 1.72 | 0.0979 |
| X3*X6 | 1 | 72.6875 | 49.1102 | 1.48 | 0.1513 |
| X4*X6 | 1 | -78.0000 | 49.1102 | -1.59 | 0.1248 |
| X5*X6 | 1 | -145.0000 | 49.1102 | -2.95 | 0.0068 |
| X6*X6 | 1 | 16.9125 | 39.5583 | 0.43 | 0.6727 |
| F | 1 | -72.1796 | 49.0202 | -1.47 | 0.1534 |
| FSQ | 1 | -627.2748 | 77.9623 | -8.05 | 0.0001 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 7 | 220490 | 31498.5679 | 0.41 | 0.8881 |
| X2 | 7 | 12862260 | 1837466 | 23.81 | 0.0001 |
| X3 | 7 | 8773624 | 1253375 | 16.24 | 0.0001 |
| X4 | 7 | 3425013 | 489288 | 6.34 | 0.0002 |
| X5 | 7 | 12077272 | 1725325 | 22.36 | 0.0001 |
| X6 | 7 | 14873164 | 2124738 | 27.53 | 0.0001 |

1) See footnote at bottom of Table B2.

# APPENDIX I

## Data and detailed analysis for Experiment 7

Table I1 - Treatment combinations and response data for experiment 7.

| OBS | F | FSQ | X1 | X2 | X3 | X4 | Y4 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | -1.000 | -1.000 | -1.000 | -1.000 | 2679 |
| 2 | 1 | 1 | 1.000 | 1.000 | -1.000 | -1.000 | 2532 |
| 3 | 1 | 1 | 1.000 | -1.000 | 1.000 | -1.000 | 1951 |
| 4 | 1 | 1 | 1.000 | -1.000 | -1.000 | 1.000 | 2536 |
| 5 | 1 | 1 | -1.000 | 1.000 | 1.000 | -1.000 | 3296 |
| 6 | 1 | 1 | -1.000 | 1.000 | -1.000 | 1.000 | 4642 |
| 7 | 1 | 1 | -1.000 | -1.000 | 1.000 | 1.000 | 4329 |
| 8 | 1 | 1 | 1.000 | 1.000 | 1.000 | 1.000 | 4132 |
| 9 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 10 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 11 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 3511 |
| 12 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 13 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 14 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 3940 |
| 15 | 2 | 0 | 1.000 | -1.000 | -1.000 | -1.000 | 1995 |
| 16 | 2 | 0 | -1.000 | 1.000 | -1.000 | -1.000 | 4507 |
| 17 | 2 | 0 | -1.000 | -1.000 | 1.000 | -1.000 | 3169 |
| 18 | 2 | 0 | -1.000 | -1.000 | -1.000 | 1.000 | 4342 |
| 19 | 2 | 0 | 1.000 | 1.000 | 1.000 | -1.000 | 3120 |
| 20 | 2 | 0 | 1.000 | 1.000 | -1.000 | 1.000 | 4250 |
| 21 | 2 | 0 | 1.000 | -1.000 | 1.000 | 1.000 | 3310 |
| 22 | 2 | 0 | -1.000 | 1.000 | 1.000 | 1.000 | 7037 |
| 23 | 3 | 1 | -1.712 | 0.000 | 0.000 | 0.000 | 4900 |
| 24 | 3 | 1 | 1.712 | 0.000 | 0.000 | 0.000 | 2597 |
| 25 | 3 | 1 | 0.000 | -1.712 | 0.000 | 0.000 | 2478 |
| 26 | 3 | 1 | 0.000 | 1.712 | 0.000 | 0.000 | 4412 |
| 27 | 3 | 1 | 0.000 | 0.000 | -1.712 | 0.000 | 3088 |
| 28 | 3 | 1 | 0.000 | 0.000 | 1.712 | 0.000 | 3480 |
| 29 | 3 | 1 | 0.000 | 0.000 | 0.000 | -1.712 | 2273 |
| 30 | 3 | 1 | 0.000 | 0.000 | 0.000 | 1.712 | 4502 |

Table I2 - Estimated second-order response surface for experiment 7, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 3597.0000 |
| ROOT MSE | 298.5682 |
| R-SQUARE | 0.96418121 |
| COEF OF VARIATION | 0.08300479 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 2363152 | 0.0730 | 13.25 | 0.0001 |
| LINEAR | 4 | 27344810 | 0.8452 | 76.69 | 0.0001 |
| QUADRATIC | 4 | 293134 | 0.0091 | 0]82 | 0.5339 |
| CROSSPRODUCT | 6 | 1193428 | 0.0369 | 2.23 | 0.1061 |
| TOTAL REGRESS | 16 | 31194523 | 0.9642 | 21.87 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 9 | 1158859 | 128762 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 4 | 0 | 0 | | |
| TOTAL ERROR | 13 | 1158859 | 89142.9980 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3903.1873 | 194.9209 | 20.02 | 0.0001 |
| X1 | 1 | -645.7693 | 63.8557 | -10.11 | 0.0001 |
| X2 | 1 | 572.5035 | 63.8557 | 8.97 | 0.0001 |
| X3 | 1 | 161.5645 | 63.8557 | 2.53 | 0.0251 |
| X4 | 1 | 692.7603 | 63.8557 | 10.85 | 0.0001 |
| X1*X1 | 1 | 68.4719 | 71.8900 | 0.95 | 0.3582 |
| X1*X2 | 1 | -45.0625 | 74.6421 | -0.60 | 0.5564 |
| X2*X2 | 1 | -35.0783 | 71.8900 | -0.49 | 0.6337 |
| X1*X3 | 1 | -28.8125 | 74.6421 | -0.39 | 0.7057 |
| X2*X3 | 1 | 27.9375 | 74.6421 | 0.37 | 0.7142 |
| X3*X3 | 1 | -90.0095 | 71.8900 | -1.25 | 0.2326 |
| X1*X4 | 1 | -129.3125 | 74.6421 | -1.73 | 0.1068 |
| X2*X4 | 1 | 117.6875 | 74.6421 | 1.58 | 0.1389 |
| X3*X4 | 1 | 200.9375 | 74.6421 | 2.69 | 0.0185 |
| X4*X4 | 1 | -54.6966 | 71.8900 | -0.76 | 0.4603 |
| F | 1 | 68.4291 | 69.3675 | 0.99 | 0.3419 |
| FSQ | 1 | -560.6618 | 113.6475 | -4.93 | 0.0003 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 5 | 9510988 | 1902198 | 21.34 | 0.0001 |
| X2 | 5 | 7453267 | 1490653 | 16.72 | 0.0001 |
| X3 | 5 | 1382189 | 276438 | 3.10 | 0.0464 |
| X4 | 5 | 11678658 | 2335732 | 26.20 | 0.0001 |

1) See footnote at bottom of Table B2.

APPENDIX J

Data and detailed analysis for Experiment 8

Table J1 - Treatment combinations and response data for experiment 8.

| OBS | F | FSQ | X1 | X2 | X3 | X4 | X5 | X6 | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | -1.000 | -1.000 | -1.000 | -1.000 | -1.000 | -1.000 | 15.34 | 30.21 | 1.36 | 1190 |
| 2 | 1 | 1 | 1.000 | 1.000 | -1.000 | -1.000 | -1.000 | -1.000 | 20.91 | 28.87 | 3.58 | 4104 |
| 3 | 1 | 1 | 1.000 | -1.000 | 1.000 | -1.000 | -1.000 | -1.000 | 19.58 | 28.87 | 3.58 | 3837 |
| 4 | 1 | 1 | -1.000 | 1.000 | 1.000 | -1.000 | -1.000 | -1.000 | 28.59 | 30.21 | 1.36 | 2222 |
| 5 | 1 | 1 | -1.000 | -1.000 | -1.000 | 1.000 | 1.000 | -1.000 | 15.34 | 27.99 | 1.36 | 1103 |
| 6 | 1 | 1 | -1.000 | -1.000 | -1.000 | 1.000 | -1.000 | 1.000 | 15.34 | 28.03 | 1.36 | 1105 |
| 7 | 1 | 1 | -1.000 | -1.000 | -1.000 | -1.000 | 1.000 | 1.000 | 15.34 | 45.87 | 1.36 | 1808 |
| 8 | 1 | 1 | 1.000 | 1.000 | -1.000 | 1.000 | 1.000 | -1.000 | 20.91 | 27.09 | 3.58 | 3847 |
| 9 | 1 | 1 | 1.000 | 1.000 | -1.000 | 1.000 | -1.000 | 1.000 | 20.91 | 27.55 | 3.58 | 3909 |
| 10 | 1 | 1 | 1.000 | 1.000 | -1.000 | -1.000 | 1.000 | 1.000 | 20.91 | 43.25 | 3.58 | 6146 |
| 11 | 1 | 1 | 1.000 | -1.000 | 1.000 | 1.000 | 1.000 | -1.000 | 19.58 | 27.44 | 3.58 | 3651 |
| 12 | 1 | 1 | 1.000 | -1.000 | 1.000 | 1.000 | -1.000 | 1.000 | 19.58 | 28.03 | 3.58 | 3727 |
| 13 | 1 | 1 | 1.000 | -1.000 | 1.000 | -1.000 | 1.000 | 1.000 | 19.58 | 43.73 | 3.58 | 5820 |
| 14 | 1 | 1 | -1.000 | 1.000 | 1.000 | 1.000 | 1.000 | -1.000 | 28.59 | 27.99 | 1.36 | 2058 |
| 15 | 1 | 1 | -1.000 | 1.000 | 1.000 | 1.000 | -1.000 | 1.000 | 28.59 | 28.03 | 1.36 | 2059 |
| 16 | 1 | 1 | -1.000 | 1.000 | 1.000 | -1.000 | 1.000 | 1.000 | 28.59 | 45.87 | 1.36 | 3374 |
| 17 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 18 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 19 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 20 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 21 | 1 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 21.35 | 34.13 | 2.54 | 3511 |
| 22 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 23 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 24 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 25 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 26 | 2 | 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.19 | 32.70 | 3.15 | 3940 |
| 27 | 2 | 0 | 1.000 | -1.000 | -1.000 | -1.000 | 1.000 | -1.000 | 14.87 | 37.37 | 3.58 | 3768 |
| 28 | 2 | 0 | -1.000 | 1.000 | -1.000 | -1.000 | 1.000 | -1.000 | 19.47 | 37.39 | 1.84 | 2535 |
| 29 | 2 | 0 | -1.000 | -1.000 | 1.000 | -1.000 | 1.000 | -1.000 | 21.48 | 37.39 | 1.84 | 2793 |
| 30 | 2 | 0 | 1.000 | 1.000 | 1.000 | -1.000 | 1.000 | -1.000 | 26.33 | 37.37 | 3.58 | 6676 |
| 31 | 2 | 0 | 1.000 | -1.000 | -1.000 | 1.000 | -1.000 | -1.000 | 14.87 | 21.57 | 3.58 | 2186 |
| 32 | 2 | 0 | 1.000 | -1.000 | -1.000 | 1.000 | 1.000 | 1.000 | 14.87 | 36.30 | 3.58 | 3677 |
| 33 | 2 | 0 | 1.000 | -1.000 | -1.000 | -1.000 | -1.000 | 1.000 | 14.87 | 37.06 | 3.58 | 3747 |
| 34 | 2 | 0 | -1.000 | 1.000 | -1.000 | 1.000 | -1.000 | -1.000 | 19.47 | 22.07 | 1.84 | 1503 |
| 35 | 2 | 0 | -1.000 | 1.000 | -1.000 | 1.000 | 1.000 | 1.000 | 19.47 | 36.75 | 1.84 | 2502 |
| 36 | 2 | 0 | -1.000 | 1.000 | -1.000 | -1.000 | -1.000 | 1.000 | 19.47 | 37.54 | 1.84 | 2550 |
| 37 | 2 | 0 | -1.000 | -1.000 | 1.000 | 1.000 | -1.000 | -1.000 | 21.48 | 22.07 | 1.84 | 1660 |
| 38 | 2 | 0 | -1.000 | -1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 21.48 | 36.75 | 1.84 | 2763 |
| 39 | 2 | 0 | -1.000 | -1.000 | 1.000 | -1.000 | -1.000 | 1.000 | 21.48 | 37.54 | 1.84 | 2811 |
| 40 | 2 | 0 | 1.000 | 1.000 | 1.000 | 1.000 | -1.000 | -1.000 | 26.33 | 21.57 | 3.58 | 3864 |
| 41 | 2 | 0 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 26.33 | 36.30 | 3.58 | 6500 |
| 42 | 2 | 0 | 1.000 | 1.000 | 1.000 | -1.000 | -1.000 | 1.000 | 26.33 | 37.06 | 3.58 | 6625 |
| 43 | 3 | 1 | -2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.35 | 28.46 | 1.36 | 1502 |
| 44 | 3 | 1 | 2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.53 | 28.17 | 3.76 | 4131 |
| 45 | 3 | 1 | 0.000 | -2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 13.69 | 28.23 | 3.15 | 2310 |
| 46 | 3 | 1 | 0.000 | 2.228 | 0.000 | 0.000 | 0.000 | 0.000 | 24.93 | 28.23 | 3.15 | 4213 |
| 47 | 3 | 1 | 0.000 | 0.000 | -2.228 | 0.000 | 0.000 | 0.000 | 13.46 | 28.23 | 3.15 | 2273 |
| 48 | 3 | 1 | 0.000 | 0.000 | 2.228 | 0.000 | 0.000 | 0.000 | 26.66 | 28.23 | 3.15 | 4502 |
| 49 | 3 | 1 | 0.000 | 0.000 | 0.000 | -2.228 | 0.000 | 0.000 | 20.06 | 40.86 | 3.15 | 4900 |
| 50 | 3 | 1 | 0.000 | 0.000 | 0.000 | 2.228 | 0.000 | 0.000 | 20.06 | 21.43 | 3.15 | 2597 |
| 51 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | -2.228 | 0.000 | 20.06 | 20.48 | 3.15 | 2478 |
| 52 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 2.228 | 0.000 | 20.06 | 36.84 | 3.15 | 4412 |
| 53 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | -2.228 | 20.06 | 18.94 | 3.15 | 2273 |
| 54 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.228 | 20.06 | 37.52 | 3.15 | 4502 |
| 55 | 3 | 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 20.06 | 28.23 | 3.15 | 3388 |

Table J2 - Estimated second-order response surface for experiment 8, variable Y1.

| | |
|---|---|
| RESPONSE MEAN | 20.6185 |
| ROOT MSE | 0.41149728 |
| R-SQUARE | 0.99471760 |
| COEF OF VARIATION | 0.01995763 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 11.7204 | 0.0146 | 34.61 | 0.0003 |
| LINEAR | 6 | 762.8842 | 0.9520 | 750.89 | 0.0001 |
| QUADRATIC | 6 | 1.3825 | 0.0017 | 1.36 | 0.2687 |
| CROSSPRODUCT | 15 | 21.1677 | 0.0264 | 8.33 | 0.0001 |
| TOTAL REGRESS | 29 | 797.1548 | 0.9947 | 162.33 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 17 | 4.2333 | 0.24901472 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 8 | -1.13687E-13 | -1.42109E-14 | | |
| TOTAL ERROR | 25 | 4.2333 | 0.16933001 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 21.5834 | 0.20250849 | 106.58 | 0.0001 |
| X1 | 1 | -0.29476649 | 0.06354993 | -4.64 | 0.0001 |
| X2 | 1 | 2.8898 | 0.06354993 | 45.47 | 0.0001 |
| X3 | 1 | 3.1237 | 0.06354993 | 49.15 | 0.0001 |
| X4 | 1 | 1.69467E-16 | 0.06354993 | 0.00 | 1.0000 |
| X5 | 1 | -1.69467E-16 | 0.06354993 | -0.00 | 1.0000 |
| X6 | 1 | 1.69467E-16 | 0.06354993 | 0.00 | 1.0000 |
| X1*X1 | 1 | 0.09591051 | 0.05859473 | 1.64 | 0.1142 |
| X1*X2 | 1 | 0.19375000 | 0.07274313 | 2.66 | 0.0133 |
| X2*X2 | 1 | -0.13172923 | 0.05859473 | -2.25 | 0.0336 |
| X1*X3 | 1 | -0.64125000 | 0.07274313 | -8.82 | 0.0001 |
| X2*X3 | 1 | 0.46125000 | 0.07274313 | 6.34 | 0.0001 |
| X3*X3 | 1 | 0.01935909 | 0.05859473 | 0.33 | 0.7439 |
| X1*X4 | 1 | 0 | 0.07274313 | 0.00 | 1.0000 |
| X2*X4 | 1 | 1.11022E-16 | 0.07274313 | 0.00 | 1.0000 |
| X3*X4 | 1 | 1.11022E-16 | 0.07274313 | 0.00 | 1.0000 |
| X4*X4 | 1 | 0.01935909 | 0.05859473 | 0.33 | 0.7439 |
| X1*X5 | 1 | 0 | 0.07274313 | 0.00 | 1.0000 |
| X2*X5 | 1 | 0 | 0.07274313 | 0.00 | 1.0000 |
| X3*X5 | 1 | 0 | 0.07274313 | 0.00 | 1.0000 |
| X4*X5 | 1 | 2.22045E-16 | 0.07274313 | 0.00 | 1.0000 |
| X5*X5 | 1 | 0.01935909 | 0.05859473 | 0.33 | 0.7439 |
| X1*X6 | 1 | -1.11022E-16 | 0.07274313 | -0.00 | 1.0000 |
| X2*X6 | 1 | 0 | 0.07274313 | 0.00 | 1.0000 |
| X3*X6 | 1 | 0 | 0.07274313 | 0.00 | 1.0000 |
| X4*X6 | 1 | -2.22045E-16 | 0.07274313 | -0.00 | 1.0000 |
| X5*X6 | 1 | 2.22045E-16 | 0.07274313 | 0.00 | 1.0000 |
| X6*X6 | 1 | 0.01935909 | 0.05859473 | 0.33 | 0.7439 |
| F | 1 | -0.58016535 | 0.07260989 | -7.99 | 0.0001 |
| FSQ | 1 | 0.12840608 | 0.11547956 | 1.11 | 0.2767 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 7 | 18.4564 | 2.6366 | 15.57 | 0.0001 |
| X2 | 7 | 358.9991 | 51.2856 | 302.87 | 0.0001 |
| X3 | 7 | 429.0922 | 61.2989 | 362.01 | 0.0001 |
| X4 | 7 | 0.01848362 | 0.00264052 | 0.02 | 1.0000 |
| X5 | 7 | 0.01848362 | 0.00264052 | 0.02 | 1.0000 |
| X6 | 7 | 0.01848362 | 0.00264052 | 0.02 | 1.0000 |

1) See footnote at bottom of Table B2.

Table J3 - Estimated second-order response surface for experiment 8, variable Y2.

| | |
|---|---|
| RESPONSE MEAN | 31.9842 |
| ROOT MSE | 0.76114739 |
| R-SQUARE | 0.99346138 |
| COEF OF VARIATION | 0.02379762 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 178.0563 | 0.0804 | 153.67 | 0.0001 |
| LINEAR | 6 | 2001.8261 | 0.9037 | 575.89 | 0.0001 |
| QUADRATIC | 6 | 15.2862 | 0.0069 | 4.40 | 0.0036 |
| CROSSPRODUCT | 15 | 5.4393 | 0.0025 | 0.63 | 0.8263 |
| TOTAL REGRESS | 29 | 2200.6079 | 0.9935 | 130.98 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 17 | 14.4836 | 0.85197845 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 8 | -9.09495E-13 | -1.13687E-13 | | |
| TOTAL ERROR | 25 | 14.4836 | 0.57934535 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 37.5659 | 0.37458038 | 100.29 | 0.0001 |
| X1 | 1 | -0.30805500 | 0.11754844 | -2.62 | 0.0147 |
| X2 | 1 | -0.03124406 | 0.11754844 | -0.27 | 0.7926 |
| X3 | 1 | 0.03124406 | 0.11754844 | 0.27 | 0.7926 |
| X4 | 1 | -4.3732 | 0.11754844 | -37.20 | 0.0001 |
| X5 | 1 | 3.6973 | 0.11754844 | 31.45 | 0.0001 |
| X6 | 1 | 3.8539 | 0.11754844 | 32.79 | 0.0001 |
| X1*X1 | 1 | -0.17651311 | 0.10838279 | -1.63 | 0.1159 |
| X1*X2 | 1 | -0.04093750 | 0.13455312 | -0.30 | 0.7635 |
| X2*X2 | 1 | -0.19363645 | 0.10838279 | -1.79 | 0.0861 |
| X1*X3 | 1 | 0.04093750 | 0.13455312 | 0.30 | 0.7635 |
| X2*X3 | 1 | 0.20218750 | 0.13455312 | 1.50 | 0.1455 |
| X3*X3 | 1 | -0.19363645 | 0.10838279 | -1.79 | 0.0861 |
| X1*X4 | 1 | 0.14406250 | 0.13455312 | 1.07 | 0.2945 |
| X2*X4 | 1 | -0.01093750 | 0.13455312 | -0.08 | 0.9359 |
| X3*X4 | 1 | 0.01093750 | 0.13455312 | 0.08 | 0.9359 |
| X4*X4 | 1 | 0.39359348 | 0.10838279 | 3.63 | 0.0013 |
| X1*X5 | 1 | -0.06343750 | 0.13455312 | -0.47 | 0.6414 |
| X2*X5 | 1 | -0.01093750 | 0.13455312 | -0.08 | 0.9359 |
| X3*X5 | 1 | 0.01093750 | 0.13455312 | 0.08 | 0.9359 |
| X4*X5 | 1 | -0.09968750 | 0.13455312 | -0.74 | 0.4657 |
| X5*X5 | 1 | -0.10701248 | 0.10838279 | -0.99 | 0.3329 |
| X1*X6 | 1 | -0.06031250 | 0.13455312 | -0.45 | 0.6578 |
| X2*X6 | 1 | -0.01906250 | 0.13455312 | -0.14 | 0.8885 |
| X3*X6 | 1 | 0.01906250 | 0.13455312 | 0.14 | 0.8885 |
| X4*X6 | 1 | -0.00906250 | 0.13455312 | -0.07 | 0.9468 |
| X5*X6 | 1 | 0.29343750 | 0.13455312 | 2.18 | 0.0388 |
| X6*X6 | 1 | -0.19363645 | 0.10838279 | -1.79 | 0.0861 |
| F | 1 | -2.0417 | 0.13430667 | -15.20 | 0.0001 |
| FSQ | 1 | -2.3236 | 0.21360280 | -10.88 | 0.0001 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 7 | 6.5321 | 0.93315373 | 1.61 | 0.1785 |
| X2 | 7 | 3.2712 | 0.46731800 | 0.81 | 0.5898 |
| X3 | 7 | 3.2712 | 0.46731800 | 0.81 | 0.5898 |
| X4 | 7 | 810.5057 | 115.7865 | 199.86 | 0.0001 |
| X5 | 7 | 576.9296 | 82.4185 | 142.26 | 0.0001 |
| X6 | 7 | 627.4843 | 89.6406 | 154.73 | 0.0001 |

1) See footnote at bottom of Table B2.

Table J4 - Estimated second-order response surface for experiment 8, variable Y3.

| | |
|---|---|
| RESPONSE MEAN | 2.7473 |
| ROOT MSE | 0.26948215 |
| R-SQUARE | 0.95325208 |
| COEF OF VARIATION | 0.09809079 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 2.7870 | 0.0718 | 19.19 | 0.0808 |
| LINEAR | 6 | 32.6993 | 0.8420 | 75.05 | 0.0001 |
| QUADRATIC | 6 | 1.0737 | 0.0276 | 2.46 | 0.0520 |
| CROSSPRODUCT | 15 | 0.46080000 | 0.0119 | 0.42 | 0.9568 |
| TOTAL REGRESS | 29 | 37.0208 | 0.9533 | 17.58 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 17 | 1.8155 | 0.10679504 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 8 | -7.10543E-15 | -8.88178E-16 | | |
| TOTAL ERROR | 25 | 1.8155 | 0.07262063 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 2.4281 | 0.13261916 | 18.31 | 0.0001 |
| X1 | 1 | 0.88311458 | 0.04161770 | 21.22 | 0.0001 |
| X2 | 1 | 0 | 0.04161770 | 0.00 | 1.0000 |
| X3 | 1 | 2.64793E-17 | 0.04161770 | 0.00 | 1.0000 |
| X4 | 1 | 0 | 0.04161770 | 0.00 | 1.0000 |
| X5 | 1 | 0 | 0.04161770 | 0.00 | 1.0000 |
| X6 | 1 | 0 | 0.04161770 | 0.00 | 1.0000 |
| X1*X1 | 1 | -0.13980921 | 0.03837263 | -3.64 | 0.0012 |
| X1*X2 | 1 | 3.46945E-17 | 0.04763816 | 0.00 | 1.0000 |
| X2*X2 | 1 | -0.02095307 | 0.03837263 | -0.55 | 0.5899 |
| X1*X3 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X2*X3 | 1 | -0.12000000 | 0.04763816 | -2.52 | 0.0185 |
| X3*X3 | 1 | -0.02095307 | 0.03837263 | -0.55 | 0.5899 |
| X1*X4 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X2*X4 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X3*X4 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X4*X4 | 1 | -0.02095307 | 0.03837263 | -0.55 | 0.5899 |
| X1*X5 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X2*X5 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X3*X5 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X4*X5 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X5*X5 | 1 | -0.02095307 | 0.03837263 | -0.55 | 0.5899 |
| X1*X6 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X2*X6 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X3*X6 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X4*X6 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X5*X6 | 1 | 0 | 0.04763816 | 0.00 | 1.0000 |
| X6*X6 | 1 | -0.02095307 | 0.03837263 | -0.55 | 0.5899 |
| F | 1 | 0.28650034 | 0.04755091 | 6.03 | 0.0001 |
| FSQ | 1 | -0.04159489 | 0.07562549 | -0.55 | 0.5872 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 7 | 33.6633 | 4.8090 | 66.22 | 0.0001 |
| X2 | 7 | 0.48245270 | 0.06892181 | 0.95 | 0.4880 |
| X3 | 7 | 0.48245270 | 0.06892181 | 0.95 | 0.4880 |
| X4 | 7 | 0.02165270 | 0.00309324 | 0.04 | 1.0000 |
| X5 | 7 | 0.02165270 | 0.00309324 | 0.04 | 1.0000 |
| X6 | 7 | 0.02165270 | 0.00309324 | 0.04 | 1.0000 |

1) See footnote at bottom of Table B2.

Table J5 - Estimated second-order response surface for experiment 8, variable Y4.

| | |
|---|---|
| RESPONSE MEAN | 3397.3818 |
| ROOT MSE | 351.8185 |
| R-SQUARE | 0.96832552 |
| COEF OF VARIATION | 0.10355576 |

| REGRESSION | DF | TYPE I SS | R-SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| COVARIATE | 2 | 1705333 | 0.0175 | 6.89 | 0.0001 |
| LINEAR | 6 | 85810457 | 0.8784 | 115.55 | 0.0001 |
| QUADRATIC | 6 | 2130833 | 0.0218 | 2.87 | 0.0288 |
| CROSSPRODUCT | 15 | 4952936 | 0.0507 | 2.67 | 0.0146 |
| TOTAL REGRESS | 29 | 94599559 | 0.9683 | 26.35 | 0.0001 |

| RESIDUAL | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| LACK OF FIT | 17 | 3094406 | 182024 | 9999.990 | 0.0001 |
| PURE ERROR 1) | 8 | 0 | 0 | | |
| TOTAL ERROR | 25 | 3094406 | 123776 | | |

| PARAMETER | DF | ESTIMATE | STD DEV | T-RATIO | PROB |
|---|---|---|---|---|---|
| INTERCEPT | 1 | 3773.2471 | 173.1390 | 21.79 | 0.0001 |
| X1 | 1 | 1047.1629 | 54.3334 | 19.27 | 0.0001 |
| X2 | 1 | 454.7772 | 54.3334 | 8.37 | 0.0001 |
| X3 | 1 | 470.4786 | 54.3334 | 8.66 | 0.0001 |
| X4 | 1 | -453.7087 | 54.3334 | -8.35 | 0.0001 |
| X5 | 1 | 387.1152 | 54.3334 | 7.12 | 0.0001 |
| X6 | 1 | 407.6566 | 54.3334 | 7.50 | 0.0001 |
| X1*X1 | 1 | -169.3868 | 50.0968 | -3.38 | 0.0024 |
| X1*X2 | 1 | 240.2500 | 62.1933 | 3.86 | 0.0007 |
| X2*X2 | 1 | -79.7411 | 50.0968 | -1.59 | 0.1240 |
| X1*X3 | 1 | 121.0000 | 62.1933 | 1.95 | 0.0630 |
| X2*X3 | 1 | -68.6250 | 62.1933 | -1.10 | 0.2804 |
| X3*X3 | 1 | -54.3582 | 50.0968 | -1.09 | 0.2882 |
| X1*X4 | 1 | -151.0000 | 62.1933 | -2.43 | 0.0227 |
| X2*X4 | 1 | -65.2500 | 62.1933 | -1.05 | 0.3041 |
| X3*X4 | 1 | -58.1250 | 62.1933 | -0.93 | 0.3589 |
| X4*X4 | 1 | 18.3656 | 50.0968 | 0.37 | 0.7170 |
| X1*X5 | 1 | 132.8125 | 62.1933 | 2.14 | 0.0427 |
| X2*X5 | 1 | 52.5625 | 62.1933 | 0.85 | 0.4060 |
| X3*X5 | 1 | 54.3125 | 62.1933 | 0.87 | 0.3908 |
| X4*X5 | 1 | 7.9375 | 62.1933 | 0.13 | 0.8995 |
| X5*X5 | 1 | -42.7748 | 50.0968 | -0.85 | 0.4013 |
| X1*X6 | 1 | 134.6875 | 62.1933 | 2.17 | 0.0401 |
| X2*X6 | 1 | 49.5625 | 62.1933 | 0.80 | 0.4330 |
| X3*X6 | 1 | 53.4375 | 62.1933 | 0.86 | 0.3984 |
| X4*X6 | 1 | 19.1875 | 62.1933 | 0.31 | 0.7602 |
| X5*X6 | 1 | 6.0000 | 62.1933 | 0.10 | 0.9239 |
| X6*X6 | 1 | -54.3582 | 50.0968 | -1.09 | 0.2882 |
| F | 1 | 65.1873 | 62.0794 | 1.05 | 0.3037 |
| FSQ | 1 | -332.1936 | 98.7317 | -3.36 | 0.0025 |

| FACTOR | DF | SS | MEAN SQUARE | F-RATIO | PROB |
|---|---|---|---|---|---|
| X1 | 7 | 51581324 | 7368761 | 59.53 | 0.0001 |
| X2 | 7 | 11286243 | 1612320 | 13.03 | 0.0001 |
| X3 | 7 | 10339589 | 1477084 | 11.93 | 0.0001 |
| X4 | 7 | 9635358 | 1376480 | 11.12 | 0.0001 |
| X5 | 7 | 7123913 | 1017702 | 8.22 | 0.0001 |
| X6 | 7 | 7876903 | 1125272 | 9.09 | 0.0001 |

1) See footnote at bottom of Table B2.

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